FINANCIAL CONTRACTING AS BEHAVIOR TOWARDS RISK: THE CORPORATE FINANCE OF BUSINESS CYCLES

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Abstract

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This paper describes a parsimonious macro finance model where contracts are the mechanism by which differentially risk averse bondholders and stockholders resolve a conflict of interest problem and confront the risks associated with future investment decisions and financing decisions of a representative firm/economy. In resolving this conflict of interest problem, the interrelated covenants in the bond contract shape certain stylized financial facts of business cycles ignored in Classical and Keynesian models. A change in risk aversion or perception of risk of investors have the representative firm adjusting both sides of their balance sheet. The asset adjustments potentially cause business cycles while the financial adjustments offset the effects of the asset/operating risk on the market valuations of bonds. The model set-up includes 2 agents (bondholders and stockholders), 2 decisions (a production/investment decision and a financing decision), and 2 equilibrium conditions (market values equal economic book value for both bonds and stocks). In this 2x2x2 set-up the contract constrained manager of the firm makes investment decisions (generating operating income and operating risk) that conforms to the risk aversion of stockholders as reflected in stock prices. The firm then makes financing decisions (generating financial risk) to offset any effect of a change in operating risk on the valuation of bonds. A cyclical expansion is set in motion by a shock that reduces risk aversion and the required rate of return of equity investors thereby increasing stock valuations relative to their economic book valuations. The representative firm responds to this arbitrage opportunity by purchasing real assets at economic book value and financing the expansion with appreciated equity. Operating income and operating risk increase with the expansion in real capital. Bond valuations fall with the increase in operating risk. However, the decision to finance the expansion with equity reduces financial leverage and financial risk thereby restoring the market value of bonds towards their initial valuation before the external shock. Theoretically, the expansion continues until market valuations for both bonds and stocks equal their respective economic book valuations thereby eliminating arbitrage profits. The model is non-traditional in that: i) contract constrained managers work for both bondholders and stockholders; ii) equilibrium in both the product/factor market and financial markets is described by a no-arbitrage condition of market value equals economic book value for both bonds and stocks; and iii) real capital investment generates both expected operating income, $\bar{X}$, and operating risk, $\sigma(X)$. A similar form or risk and return sharing is shown to occur between more risk averse mature and experienced workers and less risk averse young apprentice workers. Older and more experienced risk averse workers like bondholders opt for the safety of seniority arrangements and pay for it with a wage rate below their marginal product. Young less experienced and less risk averse workers like stockholders require a wage higher than their marginal product to accept the increased variability in their employment that come from granting seniority to the more mature workers.
I. Introduction and Literature Review

Classical economic theory asserts that in market based financial systems the goal of the manager is to maximize the market value of their firm’s equity shares. According to this view the manager’s loyalty is unmistakably to the shareholders who in turn hire and on occasion fire them. What about other stakeholders such as bondholders, bank lenders, and workers who also have substantial investments in the firm? Are their interests and welfare to be ignored? The answer of course is No. These other stakeholders are assumed to protect themselves with elaborate contracts which constrain the decisions of managers in their goal to maximize the market value of their firm’s equity. This paper is concerned with the structuring of these contracts and how they shape the relationship between capital investment decisions and financing decisions of firms over the economy-wide business cycle.

Financial economists, labor economists, legal scholars, and organizational theorists have long been interested in the nature and significance of contracts in general and financial and labor contracts in particular. Theoretical work in financial economics has focused on the conditions under which debt emerges as an optimal security design for external investors when the investment returns of the firm are opaque. It has also been concerned about when and under what conditions it is optimal to transfer control of the firm from equity investors to debt investors. Applied work takes debt as given and analyzes certain features of the negotiated debt contract that affect the market valuation of debt and equity securities. An important impetus to early-applied research on financial contracting in the U.S. was the significant study carried out by the American Bar Foundation published under the title Commentaries on Model Debenture Indenture Provisions. This study presented a detailed taxonomy and user guide for the set of protective covenants the American Bar Foundation thought represented the best practices of financial contracting. The objective of the study was to provide a template for practicing lawyers writing trust indentures for new issues of corporate debt. On the basis of this study and the Jensen and Meckling (1976) “nexus of contracts” view of the firm, Smith and Warner (1979) formulated the so-called Costly Contracting Hypothesis (CCH). This hypothesis states that because restrictive covenants (e.g., restrictions on
leverage, distributions to shareholders, working capital requirements, collateral, asset sales, and others) in debt contracts reduces the flexibility of managers, they are costly to the firm. If these covenants are costly they must provide some benefits in the form of a reduction in the agency costs of debt financing thereby reducing the required rate of return of debt investors.\(^1\) Roberts and Viscione (1986), Bradley and Roberts (2015), Chava, Kumar, and Warga (2004), Reisel (2004), and Nini and Smith (2009) empirically test this hypothesis and fail to reject the CCH.

A large program of research on financial contracting has followed the CCH. One direction this research has taken is represented in Garleanu and Zwiebel (2008) and Dichev and Skinner (2002) who theoretically and empirically analyze the design and renegotiation features in debt covenants. Related research by Chava and Roberts (2008), Nini, Smith, and Sufi (2009), Lowery and Wardlaw (2011), and the review by Roberts and Sufi (2009) analyze the characteristics of firms that cause lenders to require more (or less) covenant protection and how covenants and covenant violation impact the real decisions of firms. For example, Chava and Roberts (2008) provide evidence that covenant violation cause lenders to exert pressure on borrowers to reduce their real investment expenditures or face an acceleration of principal repayment. When covenants are set tightly, creditors are able to participate in the active management of the firm.

Much of the above empirical work on covenants and the CCH have been micro-finance oriented and carried out using cross-sectional and panel regression tests. There has been some inter-temporal theoretical and empirical work by Bradley and Roberts (2015) and Dell’Ariccia and Marquez (2006) linking covenants and collateral requirements to macroeconomic variables such as GDP and interest rates over time. Macroeconomic variables in these models are rhs variables that shape the nature and significance of covenants. In our model, the direction of causation is the opposite in that covenants shape the investment and financing decisions of non-financial firms that occasionally result in business cycles.

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\(^1\) Other important contributions to this literature include Myers and Majluf (1984), Gale and Hellwig (1985), Aghion and Bolton (1992), Hart and Moore (1994), the comprehensive review of the literature by Hart (2001), and Albuquerque and Hopenhayn (2004) among others.
More closely related to our work are papers by Covas and Den Haan (2011), Jermann and Quadrini (2012), and Begenau and Salomao (2018). While these authors do not study the role of covenants in the financing decisions of firms, they do study the relationship between new issues of debt and equity securities of nonfinancial firms to the economy-wide business cycle. The main findings for Jermann and Quadrini using the Federal Reserve Flow of Funds data are that equity payouts (i.e., dividends and share repurchases) are procyclical while debt payouts are countercyclical. The Covas and Den Haan and Begenau and Salomao studies compares the cyclical pattern of external financing for large and small Compustat firms. They find that small firms issue both debt and equity securities in cyclical expansion; i.e., external financing is procyclical for small firms. Their more surprising result is that large firm’s issue new debt to partly finance higher dividends and equity share repurchases. Our study on the other hand concerns the cyclical pattern of financial leverage (defined to be the ratio of long-term debt to total assets or equity) and not the cyclical pattern of new issues and redemptions of debt and equity securities. We find our measures of financial leverage are countercyclical which is consistent with the theory described in Section II.

In this paper we move in a different direction from the previous literature and develop a model where the changing market valuations of debt and equity securities interact with the covenants described in the Commentaries to shape the future production-investment decisions and financing decisions of a representative firm. The production-investment decisions that reflect asset adjustments of our representative firm cause business cycles. Financing adjustments depend on the operating risk associated with the asset adjustments. The covenants in this paper are designed to avoid a conflict of interest problem between bondholders and stockholders as the representative firm passes through different stages of the business cycle. More risk averse bondholders favor a relatively safe business strategy (low amplitude business cycles) that enables the firm to pay the promised interest and principal according to the terms of the bond contract. Less risk averse stockholders favor a more speculative business strategy (high amplitude business cycles) because of the call option feature embedded in the equity contract. In resolving this conflict of interest problem these covenants will end-up shaping certain financial facts of
business cycles that are ignored in Classical and Keynesian models. Towards this end, we develop a model for a representative debt and equity financed firm in which an upfront debt contract with interrelated covenants emerges as a cooperative solution to the conflict of interest problem between bondholders and stockholders. Moreover the contract solution to confronting the unknown risks of the future and resolving the conflict of interest problem in this model produces a stable product market and financial market equilibrium for the economy. Dark matter may or may not hold the universe together, but it is contracts that hold this model economy together. The no-arbitrage equilibrium takes the form of an equality between the market value and economic book values for both debt and equity securities. The interaction between the market and book valuations of debt and equity securities on the one hand, and the investment and contract-induced financing decisions of the representative firm in this model is presented in Section II. Section III presents a numerical stylized example that characterizes one possible set of linked real world covenants from the Commentaries that resolves the conflict of interest problem and supports the production-investment equilibrium and the financial market equilibrium described in Section II. Section IV provides some preliminary evidence that fails to reject the investment and financing predictions of the model in section II. Section V presents an alternative interpretation of the model as the competition between small young firms and large mature firms for capital resources over the business cycle. It is at this point that our work comes closest to that of Covas and Den Haan (2011) and Begemau and Salomao (2018). This section also applies the risk and return sharing model of Section II to the labor market. Both of these topics represent directions for future research. Finally, Section VI concludes with a summary of the main results of the paper.

II. Balance Sheet Adjustments for a Debt and Equity Financed Firm With Covenants

To motivate and simplify the conflict of interest problem between bondholders B and stockholders S for which covenants are one solution, we assume that each individual buy and hold investor in the economy

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2 Early versions of this model for the banking sector and its relationship to the Basel Accord on risk-based capital requirements are found in Krainer (2002, and 2014).
holds either a debt security or an equity security on the representative firm but not both. This arrangement of investor clienteles in a purely private and risky economy can be rationalized in many different ways. For example, Stiglitz (1972) argues that different expectations on return distributions can lead to investors splitting up into bondholders (pessimists) and stockholders (optimists). Alternatively Townsend (1979) has investors separating into stockholders (insiders) and bondholders (outsiders) when there is asymmetric information on firm returns and costly state verification. The separation of investors into bondholder and stockholder clienteles sets up the conflict of interest problem emphasized in this paper.

To present the model in the simplest and clearest possible way the two ownership structures for the firm/economy are pictured as an Edgeworth-Bowley box diagram in Figure 1. The assets (and financing) of the firm is represented by the lower and upper horizontal sides of the box and are valued at current cost or economic book value of the assets, $K$. The left and right vertical sides of the box measure the expected income, $\tilde{X}$, generated on the assets of the firm. The diagonal line in the box represents the proportional sharing rule between investors when the firm is financed solely with unlevered equity. Point $Z$ below the diagonal line reflects the sharing rule between investors when the firm is financed with both debt and levered equity. At point $Z$ in the diagram bondholders finance $K(b)$ of the assets of the firm and stockholders finance $K(s)$. For their respective investments, bondholders get $\tilde{X}(b)$ of expected income generated on those assets and stockholders get $\tilde{X}(s)$. Note that $Z$ lies below the diagonal line in the figure indicating that the expected yield on bonds, $R(b)=\tilde{X}(b)/K(b)$, is less than the expected yield on levered stocks, $R(s)=\tilde{X}(s)/K(s)$. This difference in yield measures the premium stockholders require for holding the risky levered equity security relative to the safer debt.

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3 In the Internet Appendix we offer a simple numerical example of how an all equity financed firm started by an initial entrepreneur can be transformed into a debt and levered equity firm when the next generation of investor, B and S, differ in terms of their risk aversion.
Next, we assume that \( R(b) \) and \( R(s) \) are perceived to be constant in the “small” neighborhood around the sharing point \( Z \) in Figure 1 suggesting that both investors are price-takers in the capital market. For small variations in the bondholders’ investment of \( K(b) \) in the firm, the expected required money income on debt will be:

\[
(1) \quad \overline{X}(b) = R(b)K(b)
\]

The expected money income, \( \overline{X} \), generated on the fixed amount of assets, \( K \), invested in the firm is:

\[
(2) \quad \overline{X} = R[K(b) + K(s)]
\]

where

\( R = \) the expected internal rate of return earned on the assets of the firm.

The expected (from the perspective of bondholders) residual income going to the levered equity is then:

\[
(3) \quad \overline{X}(s) = \overline{X} - \overline{X}(b)
\]
Putting (2) into (3) and then dividing the result into (1) gives the following concave relationship between financial leverage, $K(b)/K(s)$, and the distribution of firm income between bondholders and stockholders.

$$\frac{X(b)}{X(s)} = \frac{K(b) / K(s)}{R / R(b) + \{[R - R(b)] / R(b)\}K(b) / K(s)} > 0$$

The linear approximation to (4) is given by the dd schedule in Figure 2 below. Every point along the dd schedule represents varying combinations of $K(b)/K(s)$ and $X(b)/X(s)$ for which $R(b)$ from equation (1) is a constant in the small neighborhood of $Z$ in Figure 1. Note that equation (4) also indicates that non-intersecting dd schedules with a higher (or lower) embedded constant $R(b)$ lie above (or below) the schedule shown in Figure 2.

**Figure 2**

Financial Market and Product Market Equilibrium
For Debt and Equity Financed Firms

$X(b)/X(s)$

$K(b)/K(s)$
At this point it will be useful to give a market valuation interpretation to the dd schedule presented in Figure 2. Towards this end think of $R(b)$ as representing two rates of return on debt which must be equal in equilibrium. One is an expected yield, $R(b, ER)$ delivered to bondholders by the operating decisions and financing decisions of the firm; and the second is a required yield, $R(b, RR)$, of bond investors that depends on their time preference, risk perceptions, and risk aversion. For analytical convenience and ease of presentation we assume that bonds are perpetuities so that the market value of one bond, $P(b)$, when there are $N(b)$ bonds outstanding is given by:

\[
(5) \quad P(b) = \frac{\bar{X}(b)}{R(b, RR)} \cdot \frac{1}{N(b)}
\]

Error! Bookmark not defined. Multiplying the numerator of the rhs of (5) by $K(b)/K(b)=1$ and defining the expected yield on bonds to be $R(b, ER) = \bar{X}(b)/K(b)$ results in the following market price for one bond.

\[
(6) \quad P(b) = \frac{R(b, ER)}{R(b, RR)} \cdot \frac{K(b)}{N(b)}
\]

Equation (6) says that the market price of one bond equals the book value of one bond multiplied by $R(b, ER)/R(b, RR)$, a Q-ratio for bonds. Along an equilibrium dd schedule in Figure 2 we now have $R(b, ER) = R(b, RR)$, and the market value of one bond equals the book value of one bond; namely, $P(b) = K(b)/N(b)$. In equilibrium, bonds are zero NPV investments.

The equity market or ee schedule in Figure 2 is derived in the same way as the dd schedule. To see this let $\bar{X}(s)$ be the income required by equity investors for small variations of $K(s)$ in the neighborhood of point $Z$ in Figure 1 where $R(s)$ is a constant. When this is the case:

\[
(7) \quad \bar{X}(s) = R(s)K(s)
\]

The total expected income generated on the firm’s assets is again:

\[
(2) \quad \bar{X} = R[K(b) + K(s)]
\]

From the perspective of equity investors the expected income available for debt investors is a residual or equation (2) minus equation (7).
(8) \( \bar{X}(b) = X - \bar{X}(s) \)

Substituting (2) into (8) and dividing the result by (7) yields the following,

(9) \[
\frac{\bar{X}(b)}{\bar{X}(s)} = \frac{R - R(s)}{R(s)} + \frac{R}{R(s)} \left[ \frac{K(b)}{K(s)} \right] > 0
\]

Error! Bookmark not defined.

or the equation for the ee schedule in Figure 2. The ee schedule represents the combinations of \( K(b)/K(s) \) and \( \bar{X}(b)/\bar{X}(s) \) for which the rate of return on stock, \( R(s) \), is a constant in the small neighborhood of point \( Z \) in Figure 1. Equation (9) also indicates that ee schedules with higher (or lower) constant yields \( R(s) \) lie to the right (or left) of the reference ee schedule in Figure 2.

To get market valuations for equity shares, we again think of \( R(s) \) as two yields: i) an IRR or expected yield \( R(s, ER) \), and ii) a required yield \( R(s, RR) \) of equity investors. The expected yield \( R(s, ER) \) is delivered to equity investors by the production-investment decisions and the financing decisions of the firm, i.e., those decisions that determine \( K, K(b), K(s) \) and \( \bar{X} \). The required yield \( R(s, RR) \) depends on the time preference and risk aversion of equity investors along with their estimate of the operating risk generated on the firm’s productive capital. Equilibrium in the stock market requires \( R(s, ER) = R(s, RR) \).

The price of one share of stock (ignoring growth opportunities) when there are \( N(s) \) shares outstanding is:

(10) \[
P(s) = \frac{\bar{X}(s)}{R(s, RR)} \cdot \frac{1}{N(s)}
\]

Multiplying the numerator of the rhs of (10) by \( K(s)/K(s) \) and defining \( R(s, ER) = \bar{X}(s)/K(s) \) yields the following.

(11a) \[
P(s) = \frac{\bar{X}(s)}{R(s, RR)} \cdot \frac{K(s)}{N(s)}
\]

or

(11b) \[
P(s) = \frac{R(s, ER)}{R(s, RR)} \cdot \frac{K(s)}{N(s)}
\]

The ratio \( R(s, ER)/R(s, RR) \) is a Q-ratio for stock. Equation (11) says that equity investors require \( R(s, RR) \) as a result of their time preference, risk aversion, and their assessment of operating risk.
Managers in turn deliver $R(s, ER)$ with their production-investment decisions and their financing decisions. An equilibrium ee schedule in Figure 2 is one where $R(s) = R(s, ER) = R(s, RR)$. This implies from (11) that in equilibrium stocks are zero NPV investments so that $P(s) - K(s)/N(s) = 0$ everywhere along an equilibrium ee schedule. What is the meaning of the (+) and (-) signs around the ee schedule in Figure 2? They indicate the direction of change in rates of return and market valuations of equity securities due to the reciprocal nature of percentage rates of return and market valuations. The same interpretation would apply to the (+) and (-) on both sides of the dd schedule in Figure 2.

In Figure 2, it can be seen that the ee schedule intersects the dd schedule from below at point $z$ implying the slope of ee exceeds the slope of dd. It can be shown that this simply follows from the assumption of a positive risk premium of stock yields relative to bond yields. Even more simply this follows from the fact that dd in (4) is a concave function with a zero intercept in figure 2 while the ee schedule (9) is linear with a negative intercept. It is also the case at point $z$ that the capital market value of the firm’s resources equals the economic book value of those resources, and the market value of the firm’s bonds and stocks equals their respective book valuations. This condition defines the production-investment equilibrium and financial market equilibrium for this economy in that the representative firm is delivering the returns that both bondholders and stockholders require.

How do firms deliver returns and operating risk to their investors? In this model firm managers make two decisions; an asset adjustment decision, and a financing decision. Both of these decisions take time to plan and then to implement. Asset adjustments involve two decisions; one is determining the composition between relatively safe assets (eg., working capital serving as collateral for debt and a means of payment for the firm to adjust assets and liabilities) and risky assets, and the second is determining the level of assets. These asset adjustment decision generates expected operating income of $\bar{X}$ and operating risk of $\sigma(X)$. We make two assumptions regarding this return and risk generating process. The first is that expected returns, $\bar{X}$, are diminishing in the level of assets. This implies the length of the Edgeworth-Bowley box in Figure 1 grows faster than the height with increased investment in risky assets.
\( (12) \quad \bar{X} = f(K) \quad f'(K) > 0 \quad f''(K) \leq 0 \)

The second assumption is less traditional and asserts that operating risk, \( \sigma(X) \), is increasing in the level of risky assets. Operating risk is endogenous in this model and linked to real capital investments.\(^4\)

\( (13) \quad \sigma(X) = g(K) \quad g'(K) > 0 \quad g''(K) \geq 0 \)

Two empirical observations support this assumption. First, there is much evidence that banks lower their credit standards when lending to firms in cyclical expansions when investment is high (see Weinburg 1995, Keeton 1999, and Berger and Udell 2003). This has led to a regulatory response in the form of the countercyclical capital buffer requiring banks to increase their tier 1 capital when loan growth exceeds GDP growth in cyclical expansions (see Demirguc-Kunt and Detragiache, 1998). More directly, Kotheri et al. 2002 and Suurmeijer et al. 2013 provide micro empirical evidence that increased volatility of future earnings follows increases in current investment for Compustat industrial firms. This micro evidence can be supplemented with some macro evidence from the U.S. nonfinancial corporate sector. Following Kothari et al. we regress the log of the standard deviation of before tax real profits computed over the 5 year period from \( t \) through \( t+4 \) on the log of current real capital expenditures (P&E, inventories and non-produced nonfinancial assets) in year \( t \) and the change in financial leverage in year \( t \). The financial leverage variable is included since finance theory predicts that increases in leverage increases the volatility of before tax profits. The results with an AR(1) transformation and Newey-West t-scores/P-values beneath the estimated coefficients are:

\[
\begin{align*}
(13a) \quad \text{Log (Stddev Profits, BT)}_{t+4} &= -4.86 + 1.38 \text{ Log (Cap Expend)}_t + 9.85 \Delta (\text{Liab/A})_t \\
& (\text{-2.17/.00}) (4.07/.00) (1.74/.09) \quad \bar{R}^2 = .80 \quad \text{AR(1) = .67}
\end{align*}
\]

All variables were deflated by the GDP deflator and obtained from Table F102 of the Federal Reserve Financial Accounts at [www.federalreserve.gov/releases/z1...b102](http://www.federalreserve.gov/releases/z1...b102). The estimated coefficient on the

\(^4\) One implication of (12) and (13) is that high (or low) rates of return on real capital are associated with low (or high) volatility of returns, a result consistent with the “low risk anomaly” literature on equity returns recently revived by Baker and Wurgler (2015).
variable of interest, log(CapExpend), is positive and statistically significant implying a 1 percent increase in investment results in a 1.38 percent increase in the standard deviation of before tax future profits. A pairwise Granger Causality test for a lag of 2 years indicates that we can reject the hypothesis that the log (CapExpend) does not Granger cause log (StddevProfits,BT) at the .01 level of significance since the F-statistic is 5.49. However we cannot reject the hypothesis at the 5 percent significance level that log (StddevProfits,BT) does not Granger cause log (CapExpend) since the F-statistic is 2.29 with a Prob of .12. Finally, the Cusum and Cusum of Squares plots (not shown here, but available on request) indicate that we cannot reject the hypothesis that the estimated coefficients are stable over the sample period. The micro and macro evidence is consistent with the assumptions in equation (13). The second decision the firm makes is a financing decision which is represented by financial leverage, K(b)/K(s). An increase in K(b)/K(s) increases financial risk which magnifies the operating risk of the firm.

**Figure 3**

Investment Adjustments that Favor Stockholders
Investment decisions and financing decisions can redistribute wealth among debt and equity investors. This is described in Figure 3 and 4 where we discuss the (+) and (-) signs around the dd and ee schedules. To see this consider some initial equilibrium like z in Figure 3 where the market value of debt and equity securities equals their respective book values and the economic book value of the productive resources of the firm. Now suppose managers rearrange the asset mix of the representative firm buying risky assets and paying for them by selling safe assets. On the assumption that risky assets (e.g., P&E, inventories, R&D, and advertising) generate higher expected returns than investments in safe assets (e.g., cash and short-term securities), this adjustment will increase the expected income, $\bar{X}$, and operating risk, $\sigma(X)$, of the firm and the residual income $\bar{X}(s)$ according to (12) and (13). This is indicated by a downward vertical movement in Figure 3. Since this is a pure asset switch with no change in $K(b)$ or $K(s)$, this adjustment results in an increase in $R(s,ER) = \bar{X}(s)/K(s) > R(s,RR)$ and consequently
P(s) > K(s)/N(s). As can be seen in zone IV of the figure the market value of stocks now exceeds the economic book value of stocks and stockholders are experiencing capital gains as indicated by the (+) sign around ee. What about bonds? On the assumption that the increase in operating risk dominates the expected return effect, risk averse bond investors will increase their required yield R(b,RR) resulting in P(b) < K(b)/N(b) so that bonds are now selling at market prices below economic book value.

Bondholders have now suffered capital losses because of the risky investment decision as indicated by the (-) sign beneath the dd schedule. We will see later that covenants in bond contracts are designed to keep the firm permanently away from points in zone IV. Similarly, a pure asset switch (with no change in financing) away from risky assets and into safe assets will push the representative firm into zone II as illustrated in Figure 4. This decision will reduce $\bar{X}$, $\sigma(X)$, and $\bar{X}(s)$ and is indicated by an upward vertical movement from the initial equilibrium at z in the figure. This asset switch that reduces $\bar{X}$ will also reduce $\bar{X}(s)/K(s) = R(s,ER)$ relative to the $R(s,RR)$ at z resulting in P(s) < K(s)/N(s). Stockholders suffer capital losses in zone II as indicated by the (-) sign above the ee schedule. In this case (and on the assumption the risk effect dominates the income effect) bondholders experience capital gains indicated by the (+) sign above the dd schedule in Figure 4. This is because the reduction in operating risk with no change in financial risk causes bondholders to reduce their required rate of return resulting in $R(b,ER)/R(b,RR) > 1$ and consequently $P(b) > K(b)/N(b)$. To control these wealth redistributions the goal of rational financial contracting is to keep the firm away from permanently settling in zones II and IV. Specific covenants designed to achieve this objective are presented below in Section III.

But what about zones I and III in Figure 2? It turns out that they too are disequilibrium zones in a competitive economy. In zone I the market values of the representative firm’s bonds and stocks exceeds the economic book value of its productive resources. The market signal in this case is that new firms or parts of existing firms can be created in the product-factor market at a cost of economic book value, and then paid for with debt and equity claims issued in the capital market for a greater value thereby producing an arbitrage profit. This arbitrage between the product-factor market and the financial capital market would eventually eliminate this discrepancy between market and book value. Moreover, at the
economy-wide level this arbitrage process of buying resources in the factor input market in zone I will cause a business cycle expansion. In zone III, the situation is reversed in that the market values of both debt and equity securities are below the economic book value of the firm’s productive resources. The market signal here is for managers to sell the firm’s productive resources back to the product-factor market at the higher book value and then use the proceeds of the sale to retire the securities issued against those resources thus producing an arbitrage profit. Hostile takeovers by private equity and other investors are mechanisms that implement this downsizing. At the economy-wide level this downsizing arbitrage would cause a business cycle recession.

The only stable no-arbitrage equilibrium for the firm/economy is given by point $z$ in Figure 2 where the capital market value of the firm’s securities is exactly equal to the economic book value of its productive resources. How can the managers of the representative firm in zones I or III find this equilibrium point? To find and grope towards this point it is only necessary for the firm managers to know the market values of its securities and the economic book value of its assets at the point in time when it makes the investment and financing decisions. With this information, the firm managers can adjust the asset and financing sides of its balance sheet in order to generate the expected rates of return the two investors require. The actual path of the approach to the equilibrium $z$ depends on whether the firm first adjusts the asset side or the financing side of its balance sheet or both simultaneously.

To see this, consider some initial product market and financial market equilibrium represented by $z$ in Figure 5. Now suppose some favorable external shock causes equity investors to re-price risk by reducing their required rate of return from $R(s, RR)$ embedded in $ee$, to $R'(s, RR)$ embedded in the $e''e''$ schedule. Without loss in generality and to keep Figure 5 relatively clean we ignore any initial changes in the required yield for bondholders resulting from the initial shock. The new equilibrium that now emerges after this shock is located at $z''$; in other words, the intersection of the original dd schedule and the new lower required equity yield schedule $e''e''$. However, the firm’s present assets and financing are still generating expected yields for equity investors of $R(s, ER)$, and that higher yield is embedded in the pre-shock $ee$ schedule. Consequently, after the shock the firm is on the borderline of zones I and IV at
point $z$. At point $z$ the market price of a bond by equation (6) is $P(b) = K(b)/N(b)$. Stocks, however, are valued by equation (11) and are now at a premium of $P(s) > K(s)/N(s)$ since $R(s, ER) > R''(s, RR)$. The first business strategy considered is one where the firm implements a financing decision and then an investment decision. The first stage financing decision is illustrated in Figure 5 as the firm moves from $z$ toward point 1 on the $e''e''$ schedule by raising equity finance and investing in cash. Eventually the financing adjustment puts the firm on the $e''e''$ schedule at point 1. At point 1 the financing decision that increases $K(s)$ relative to $X(s)$ drives down the expected yield on stock from equation (11a) so that $R''(s, ER) = R''(s, RR)$ with the result that $P(s) = K(s)/N(s)$. In eliminating the disequilibrium in the stock market when going from $z$ to 1, the firm creates a disequilibrium in the bond market of $P(b) > K(b)/N(b)$.

The financing decision that reduced financial leverage and financial risk along with the decision to hold operating risk constant by investing in cash reduces the risks confronting bondholders thereby inducing an increase in the investment quality and market valuation of bonds.\textsuperscript{5} When at point 1 the firm then implements the second stage investment decision by accumulating risky assets with the cash obtained from raising equity finance. This accumulation of risky productive assets by equation (12) generates higher expected returns moving the firm/economy away from point 1 to point 2 in the figure but at the expense of higher operating risk by equation (13). At point 2 on the dd schedule the increased operating risk generated by the firm’s investment decision just offsets the first stage reduction in financial leverage and financial risk so that $P(b) = K(b)/N(b)$. This two-step process of first reducing financial leverage and financial risk, and then increasing risky investments and generating increased operating risk continues until the firm/economy reaches point $z''$, the cyclical expansion equilibrium point. At that point the capital market provides no more arbitrage opportunities for the firm/economy. In the end stockholders get the riskier corporate assets they initially signaled through stock prices that they wanted and bondholders get the financial adjustment that exactly insulates them from this increased operating risk.

\textsuperscript{5} If the representative firm for some reason remained at point 1 in Figure 5, the dd schedule would ultimately shift down to a lower rate of return schedule reflecting the higher investment quality of the firm’s bonds. However as we will see below that when the firm expands its risky investments in going from 1 to 2 (thus increasing its operating risk) the dd schedule would then drift upward towards its original position.
The welfare of both investors has been coalesced in going through an expansion from $z$ to $z''$. It should be noted in passing that had the firm responded to the initial stock market signal at $z$ by reducing their risky corporate investments and operating risk, and increasing financial leverage and financial risk, they would have moved further away from the equilibrium at $z''$.

**Figure 5**

Financial Market and Product Market Equilibrium
For Debt and Equity Financed Firms: Expansion

In the “finance first and then invest,” the expansion adjustment path for the firm/economy would take place exclusively in zone I of Figure 5 in this example. If instead the firm makes it’s real investment decision first (financed with safe cash assets accumulated in the previous recession) and then the financial decision, the adjustment path would take place in zone IV. To see this, start again in Figure 5 at point $z$ after the favorable risk aversion shock reduces the required rate of return of stockholders producing a new equilibrium at $z''$. The two-step “invest first and finance second” is illustrated in the figure by movements...
from $z$ to 1” and then 1” to 2”. The expansion in risky investments from $z$ to 1” increases $\bar{X}$, $\bar{X}(s)$ and $R(s,ER)$ from (12), and operating risk $\sigma(X)$ from (13). The increase in operating risk reduces the investment quality and market valuation of bonds. However, a rational bond contract designed to keep the firm from remaining in zone IV now requires a financial adjustment in the form of an increase in equity finance and reduction in financial leverage and financial risk. The proceeds from the equity issue/retentions are invested in risk-free assets. This is described in the figure as a movement from 1” to 2”. At 2” the firm is back on the dd schedule where $P(b) = K(b)/N(b)$. The reduction in financial risk when reducing financial leverage from 1” to 2” exactly offsets the increase in operating risk that accompanies the expansion in real productive investment when going from to $z$ to 1”. This process of adjustment would continue until the firm reaches $z''$ at which point both a product-factor market and financial market equilibrium is attained. In the end the less risk averse stockholders again have the riskier real production-investment strategy they signaled they wanted at point $z$, but bondholders have a safer financial strategy and therefore the welfare of both investors have been coalesced during the cyclical expansion in production-investment.

The recession case is presented in Figure 6. The initial position of the firm/economy is at point $z$, the intersection of an equilibrium dd and ee schedules. Now let there be a negative taste for risk shock that raises the required yield of equity investors from $R(s,RR)$ to $R'(s,RR) > R(s,RR)$ shifting the ee schedule from ee to e’e’ with a new contract point at $z'$. The reaction in the stock market is immediate with $P(s) < K(s)/N(s)$. Suppose the firm reacts to this disequilibrium by first implementing a financial adjustment and then a production-investment adjustment. In going from $z$ to point 1 the firm retires equity with a cash payout which increases financial leverage and financial risk but eventually gets the stock market back into equilibrium so that $P(s) = K(s)/N(s)$. In going from $z$ to 1 the financing decision of increasing leverage and financial risk reduces the investment quality and market valuation of bonds so that $P(b) < K(b)/N(b)$. The second step in the adjustment process is to downsize the firm by selling productive resources back to the product/factor market holding cash which then reduces $\bar{X}$ and $\bar{X}(s)$ from
(12) and reduces operating risk $\sigma(X)$ from equation (13). The reduction in operating risk when going from point 1 to point 2 eventually offsets the increase in financial risk when going from $z$ to 1, so that at point 2 the bond market is back in equilibrium at which point $P(b) = K(b)/N(b)$. This step by step adjustment within zone III is then repeated until the firm reaches the point $z'$ where the bond and stock markets signal a no arbitrage level and structure of assets and an equilibrium financial structure. At the recession equilibrium point $z'$ bondholders end up with a larger share of corporate income in exchange for their larger share in financing a downsized and safer firm. Stockholders on the other hand have allowed their call option to expire and partially withdrawn their investment in the firm/economy. The end result is that the welfare of both groups of investors has been coalesced in the recession equilibrium. To go against the signals from the capital market by increasing real investment and increasing operating risk and reducing financial leverage (and financial risk) will take the firm/economy further from the new equilibrium at $z'$.

Figure 6
Financial Market and Product Market Equilibrium
For Debt and Equity Financed Firms: Recession
An alternative path from $z$ to $z'$ is for the firm to first implement the asset adjustment and then the financing adjustment. This path takes the firm through zone II in its adjustment from $z$ to $z'$. In this case the firm first sells risky assets back to the product/factor market (investing the proceeds in safe cash assets) which is described in Figure 6 by the upward vertical movement from $z$ to $1^*$. As the firm downsizes it reduces $\bar{X}$ and $\bar{X}(s)$ from (12) and $\sigma(X)$ from (13) which in turn increases bond valuations so that at $1^*$ $P(b) > K(b)/N(b)$. However, the market valuations of stock have fallen even further when going from $z$ to $1^*$ since expected returns $R(s,ER)$ falls with the reduction in investment. A rational financial contract now allows stockholders to withdraw their equity investment in the firm thereby increasing financial leverage and financial risk. Eventually this financing adjustment from $1^*$ to $2^*$ that increases financial risk offsets the reduction in operating risk when going from $z$ to $1^*$ and bond prices drift back to $P(b) = K(b)/N(b)$. A repeat of this two-step adjustment would eventually take the firm/economy to $z'$ at which point $P(b) = K(b)/N(b)$ and $P(s) = K(s)/N(s)$. No investor is exploited at the recession equilibrium point at $z'$.

This section has presented a model of a representative debt and equity financed firm in which there is a conflict of interest problem between bondholders and stockholders over the future investment and financing decisions of the firm. These investment and financing decisions shape future business cycles. The solution to this conflict of interest problem is some form of shared management of the firm. In bank oriented financial systems like those in Europe and Japan, this shared management takes the form of various councils and boards comprised of debtholders (including workers) and stockholders. These councils and boards formulate a mutually agreed upon business strategy and monitor its implementation by the managers. In stock market oriented financial systems like the U.S. and U.K. the shared management of the firm is implemented with various covenants in debt (and labor) contracts. In this connection it is useful to classify business decisions into two different categories: i) asset adjustments comprised of real production-investment decisions that determine the fundamental operating income and risk of the firm; and ii) financial adjustments that determine the financial risk of the firm and the
distribution of corporate income among bondholders and stockholders. According to the model in this section these balance sheet adjustments can be implemented together in a unique way by managers that results in a stable non-exploitive product/factor market and financial market equilibrium in the economy. More particularly we have shown in this section that with two types of investors, two types of business decisions (i.e., production-investment decisions and financing decisions), and two equilibrium conditions (i.e., market valuations on both debt and equity securities equal to their respective book valuations), an overall product/factor market equilibrium and financial market equilibrium both exist and both are stable. The attainment of this product/factor market and financial market equilibrium can be achieved with a simple assignment rule in the sense of Mundell (1962) that coalesces the welfare of both bondholders and stockholders through different states of the economy. That assignment rule has the firm making production-investment decisions to conform to the risk aversion of their equity investors as reflected in stock prices, and then making financing decisions that insulate the bondholders from any change in operating risk resulting from the production-investment decisions. In this model it is stock and bond prices subject to a contract constraint that guide the investment and financing decisions of the representative firm through time. In section III we study the ways in which covenants in bond contracts can help implement this assignment rule.

---

6 In some ways this line of reasoning is similar to certain arguments made in the traditional incomplete contracts literature concerning when it is optimal to transfer control of the firm’s productive assets from the entrepreneur to the outside investor. Aghion and Bolton (1992) have studied this problem and concluded that under certain circumstances it is optimal for the entrepreneur (i.e., our stockholders) to retain control in the good states of nature (our zone I) with high profits but high risk, and that outside investors (i.e., our bondholders) should take control of the firm in bad states of nature (our zone III) with low profits but low risk. Kaplan and Stromberg (2001) in a study of 213 investments by 14 venture capital partnerships provide some supporting evidence for this hypothesis. In our model managers use financial policy to offset changes in operating risk resulting from investment decisions.

7 We have chosen to describe this assignment rule in Figures 5 by having the firm adjust its assets and financing one step at a time. This was done for illustrative purposes only. What it illustrated was the effect on one investor’s market wealth when the other investor makes an optimal decision from their perspective. Of course the firm could make both decisions simultaneously in various ways. One way would be to adjust both assets and financing simultaneously in a way that keeps the firm on the dd schedule where market value equals book value for debt securities when the firm goes from z to z” in the figure. Presumably, the objective of bond covenants would be to achieve this outcome. An alternative way is to adjust both investments and financing together so that market and book values for both debt and equity securities are equalized along the expansion path in Figures 3. This case was illustrated in Krainer (1992, pp. 97-100 and 109-111).
Financial Contracting: An Example of Asset and Financing Adjustments over the Business Cycle

Financial contracts are lengthy, complicated, and costly legal documents that naturally evolve in a financially developed economy. The growth in the length and complexity of these contracts has been extraordinary. According to the historical review provided by Rodgers (1965, pp. 552 and 555), the trust indenture of the Philadelphia, Germantown, and Norristown Rail Road Company in 1833 was only three pages; in 1962 the trust indenture for the Wabash Securities Corporation was 298 pages. The American Bar Foundation’s Commentaries on Model Debenture Indenture Provisions was an attempt to simplify and shorten the bond contract. Bond covenants are cataloged and analyzed in Article 10 of the Commentaries. The negotiated covenants include: the provision of financial information for bond investors, insuring the productive assets of the company, keeping the productive capital in good working order, limitations on encumbrances and future debt issues, restrictions on dividends and other equity distributions, limitations on the disposition of and sale and leaseback of capital assets, maintenance of net working capital, and restrictions on investments. Moreover the Commentaries provide a number of specific examples of the different ways in which these static “stand-alone” covenants might be written for actual use in real world trust indentures. As such the Commentaries were meant to have an immediate impact on the practice of financial contracting.

From the perspective of the model presented in Section II, static stand-alone covenants are in general suboptimal. Recall from Section II that business decisions were subdivided into two broad categories. One category was the real production-investment decision that generates the operating income and operating risk of the firm. These decisions are reflected on the asset side of the firm’s balance sheet. The second category included the financing decisions that generate the financial risk of the firm. These decisions are reflected on the liabilities and equity side of the firm’s balance sheet. The covenant constrained objective of the firm in Section II was to adjust these two sides of the balance sheet (in response to the external shock-induced bond and stock market signal) so as to equate at the margin the market value and economic book value of both its debt and equity securities. This objective required the
firm to make its asset adjustment decision to conform to the risk aversion of its stockholders. Financing
decisions were then made to insulate bondholders from any change in operating risk accompanying the
investment decision. In other words, financing decisions must be matched to the investment decisions of
the representative firm so that changes in operating risk are offset with changes in financial risk.

In terms of the model described in Figures 4 and 5, one of the objectives of bond covenants is to
keep the firm away from the redistributive zone IV (or II) where the market value of debt is below (or
above) book value and the market value of equity is above (or below) book value.\(^8\) This objective can be
achieved by linking together two well-known covenants that are described in the Commentaries. These
covenants include restrictions on: future debt financing, investments in net working capital, and cash
payments to stockholders. The model covenant that achieves the matching of financial adjustments to
asset adjustments so necessary in keeping the firm away from zone IV can be written in two parts as:

\[
\begin{align*}
(14) \quad WC & \geq \gamma \cdot LTDebt \quad \gamma > 0 \\
\end{align*}
\]

and

\[
\begin{align*}
(15) \quad & \text{when } WC > \gamma \cdot LTDebt \quad Div \leq X(E) - [\gamma \cdot LTDebt - WC] \\
& \text{when } WC = \gamma \cdot LTDebt \quad Div \leq X(E) \\
& \text{when } WC < \gamma \cdot LTDebt \quad Div \leq X(E) - [\gamma \cdot LTDebt - WC] \quad \text{and } 0 < \gamma < 1 \\
\end{align*}
\]

where

\[
WC = \text{Actual net working capital or the difference between short-term assets and short-term}
\]

liabilities. In the numerical example presented below we ignore short-term liabilities so that net working
capital equals short-term (and safe) assets. This variable represents an investment decision.

\[\text{\textsuperscript{8} One question that arises is why purely financial arbitrage would not take the firm away from zone IV towards point z in Figures 4 and 5 thus obviating the need for costly protective covenants. Moreover, purely financial arbitrage by the firm and/or other investors would seem to be more profitable than the firm’s arbitrage with the product/factor market that occurs in zones I and III. For example, in zone I the firm can issue securities at market prices above book value to acquire productive assets in the factor market at prices equal to book value. On the other hand, in zone IV the firm can issue equity at market prices above book value and buy-up debt securities at prices below book value. The deeper the firm is in zone IV the more profitable this financial arbitrage becomes. In this context the purpose of protective covenants is to limit the magnitude of these arbitrage profits that accrue to stockholders but come at the expense of bondholders.}\]
LT Debt = Long-term funded debt containing the covenant. This variable represents a financing decision.

\( \gamma \) = A negotiated positive parameter reflecting the operating risk of the firm.

Div = The sum of cash dividends and share repurchases. This variable represents a financing decision.

\( X(E) \) = The sum of annual earnings to equity investors and new equity issues net of debt issues.

In the numerical example presented below we ignore new issues so that \( X(E) = X(s) \).

The first part of the covenant in (14) links the investment in net working capital with long-term debt. For simplicity it is assumed that the relationship is linear although there is no reason why it cannot be nonlinear. The parameter \( \gamma \) reflects the industry risk of the company. For relatively safe and stable industries \( \gamma \) will be relatively small, and the amount of collateral for bondholders in the form of net working capital will be small. The riskier the industry, the higher will be the value of \( \gamma \). For example, \( \gamma = 1 \) means that at a minimum the firm’s investment in net working capital must be as large as its long-term debt outstanding. In effect bondholders are demanding collateral in the form of short-term safe assets at least as great as the amount of long-term debt outstanding.

The second part of the covenant in (15) says that the amount of dividends (including share repurchases) the firm can pay to its shareholders depends on whether the collateral requirement in (14) is met. If the firm has a sufficient investment in net working capital to satisfy the negotiated amount required by the covenant in (14), then it is free to distribute a cash dividend at least equal to its earnings and new equity issues net of new debt issues. In this case the dividend policy is relatively unconstrained. On the other hand, if the collateral requirement in (14) is breached, then a remedy is automatically put in place in the form of a restriction on cash distributions to shareholders. The purpose of the remedy is to use the contractually created retained earnings of \( X(E) - [\gamma \text{ LTDebt} - \text{WC}] \) to rebuild the deficiency in net working capital, or, in some cases to reduce the amount of long-term debt outstanding. When \( 0 < \gamma < 1 \), the most efficient way (in terms of forgone cash distribution to shareholders) to repair any breach in (14) is to use the contractual retained earnings of \( X(E) - [\gamma \text{ LTDebt} - \text{WC}] \) to build-up the firm’s investment in
net working capital either by accumulating short-term assets or retiring short-term liabilities. To retire long-term debt to repair the breach would require a greater reduction in dividends. On the other hand when the negotiated $\gamma = 1$, there is no difference in terms of foregone dividends between accumulating net working capital or reducing long-term debt in terms of repairing any breach in (14).\(^9\)

At this point it will be useful to illustrate how this set of covenants and the model economy described in Section II would work in practice with a stylized numerical example. In this connection let the balance sheet for some hypothetical representative firm (m) initially (say at point $z$ in Figure 5) be:

<table>
<thead>
<tr>
<th>Firm(m)</th>
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<th></th>
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</thead>
<tbody>
<tr>
<td>ST Assets</td>
<td>300</td>
<td>LT Debt K(b)400</td>
</tr>
<tr>
<td>Risky Assets</td>
<td>700</td>
<td>Equity K(s) 600</td>
</tr>
<tr>
<td>Total K</td>
<td>1000</td>
<td>Total K 1000</td>
</tr>
</tbody>
</table>

It is assumed that there are $N(b) = 4$ bonds outstanding each with a book value of 100 per bond, and $N(s) = 60$ shares of stock each valued at $P(s) = 10$ per share. The specific concave return generating process indicating larger returns are earned on risky assets compared to safe assets is assumed to be:

$$X = 5(\text{ST Assets})^{\frac{1}{2}} + 15(\text{Risky Assets})^{\frac{1}{2}}$$

$$X = 5(300)^{\frac{1}{2}} + 15(700)^{\frac{1}{2}} = 483.5$$

The gross return of $X = 483.5$ is assumed to be initially divided among bondholders and stockholders in the following way:

$$X(b) = 183.5$$

$$X(s) = 300$$

\(^9\) What about the case of $\gamma > 1.0$? This would imply that the firm must have net working capital as collateral in excess of its long-term debt. If the short-term liquid assets that comprise one component of net working capital were risk-free, then $\gamma > 1.0$ would not seem to make economic sense in that for every dollar borrowed by the firm would carry a collateral requirement of more than a dollar in a risk-free asset. On the other hand if the short-term assets were risky, then it is not possible to rule out a $\gamma > 1.0$. In this case the second part of the covenant would be:

$$Div \leq \left\{ X(E) - \left[ LTDebt - \frac{1}{\gamma}WC \right] \right\}$$

when $WC < \gamma LTDebt$ When this is the case the most efficient way to repair a breach in (15) is to retire long-term debt. To comply with this covenant, by increasing the firm’s investment in net working capital, would require a greater reduction in the firm’s cash distribution to the shareholders.
Financial leverage and the distribution of corporate income between bondholders and stockholders are then \( \text{LT Debt / Equity} = \frac{400}{600} = .67 \) and \( X(b)/X(s) = \frac{183.5}{300} = .61 \). In this initial equilibrium the expected and required rates of return on bonds and stocks are assumed to be respectively \( R(b,ER) = R(b,RR) = .459 \), and \( R(s,ER) = R(s,RR) = .50 \). This initial position for firm (m) at time period \( t = 0 \) is presented in column 2 of Table 1. The negotiated covenant from (14) and (15) is assumed to be:

\[
WC \geq \gamma \text{LTDebt} \quad \gamma = .75
\]

and:

- when \( WC > \gamma \text{LTDebt} \), \( \text{Div} \leq X(E) - [\gamma \text{LTDebt} - WC] \)
- when \( WC = \gamma \text{LTDebt} \), \( \text{Div} \leq X(E) \)
- when \( WC < \gamma \text{LTDebt} \), \( \text{Div} \leq X(E) - [\gamma \text{LTDebt} - WC] \)

The balance sheet indicates that firm(m) is in compliance with the covenants in its bond contract since \( WC = \gamma \text{LTDebt} \) or \( 300 = .75(400) \), therefore this firm can pay cash dividends or repurchase stock up to a maximum given by the dividend covenant in (15), or, \( X(s) = \text{Div} = 300 \).\(^{10}\)

----------Insert Table 1 about here----------

Now suppose the economy experiences a negative taste for risk shock that increases the required rate of return of equity investors from \( R(s,RR) = .50 \) in \( t=0 \) to \( R'(s,RR) = .541 \) in \( 0 < t < 1 \). As a result of this shock the market value of one share of stock falls to:

\[
P(s) = \frac{R(s,ER)}{R'(s,RR)} \cdot \frac{K(s)}{N(s)} \quad \text{or} \quad P(s) = \frac{.500}{.541} \cdot 10 = 9.24
\]

in \( 0 < t < 1 \). Now the market value of the firm, \( N(b)P(b) + N(s)P(s) = 943.40 \) is below the assumed constant economic book value of \( K(b) + K(s) = 1000 \) as described in column 3 of Table 1. Market valuations less than economic book valuations are the market signal for managers to formulate the relatively safe operating strategy of downsizing the firm. Managers implement this strategy by reducing their investments in Risky Assets by 100 and increasing their investments in ST Assets (hoarding cash as

\(^{10}\) It is assumed that the initial balance sheet for point \( z \) is measured after the previous period earnings were distributed to stockholders in the form of cash dividends and/or stock repurchases.
the nonfinancial corporate sector did in the Great Recession) by the same amount. The balance sheet of firm (m) is now

<table>
<thead>
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<td>600</td>
<td></td>
</tr>
<tr>
<td>Total K</td>
<td>1000</td>
<td></td>
</tr>
</tbody>
</table>

The returns to the firm, bondholders, and stockholders are now (by the assumed concave return generating process) the following:

\[ X = 5(400)^{1/3} + 15(600)^{1/3} = 467.4 \]

\[ X(b) = 183.5 \]

\[ X(s) = 283.9 \]

The end result of this asset adjustment decision is that the covenant in the long-term debt contract is now relaxed since

\[ WC > \gamma LTDebt \]

or

\[ 400 > .75(400) \]

How much cash can potentially be returned to stockholders in the interim period during the process of the firm’s downsizing? According to the second part of the covenant, we now have:

\[ \text{Div} \leq X(s) + [WC - \gamma LTDebt] \]

Or

\[ \text{Div} \leq 283.9 + [400 - .75(400)] \leq 383.9 \]

Thus the maximum amount of cash that can be distributed to shareholders in the interim period is 383.9; 283.9 from earnings and 100 from ST Assets resulting from the relaxation of the bond covenant. This extra distribution to shareholders of 100 is financed by reducing firm (m)’s investment in STAssets. To a certain extent stockholders are letting their option on the underlying assets of the firm expire. If the maximum dividend is paid the balance sheet in the recession period t=1 for the downsized firm is:
The returns to the firm, bondholders, and stockholders in the recession equilibrium are now:

\[ X = 5(300)^{\frac{1}{2}} + 15(600)^{\frac{1}{2}} = 454.0 \]
\[ X(b) = 183.5 \]
\[ X(s) = 270.5 \]

Moreover the long-term debt covenant is just satisfied since

\[ WC \geq \gamma \text{LTDebt} \]

Or

\[ 300 = .75(400) \]

Therefore in the new recession equilibrium of \( t=1 \) firm (m) can pay out all of its reduced earnings of 270.5 in dividends. It can also be seen that stockholders now earn their new required rate of return of \( R(s,ER) = R'(s,RR) = .541 \); bondholders still earn \( R(b,ER) = R(b,RR) = .459 \); financial leverage in \( t=1 \) is higher, LT Debt/Equity = 400/500 = .80, compared to .67 in \( t=0 \); while the distribution of corporate income is now \( X(b)/X(s) = 183.5/270.5 = .68 \), compared to .61 in \( t=0 \). Stockholders’ relative share in corporate income falls when firm (m) implemented the safe investment strategy while bondholders share in financing the firm is now larger. The end result is that in the period \( t=1 \) recession, firm (m) generates less operating risk as a result of the downsizing investment decision. However this reduction in operating risk has been offset with an increase in financial leverage and financial risk. At this point the market value of bonds and stocks, \( N(b)P(b) + N(s)P(s) = 900 \) equals the book value of these securities, \( K(b) + K(s) = 900 \), and the book value of the productive assets. All arbitrage opportunities between the capital market and factor market are exhausted. These adjustments are described in column 4 of Table 1.

Finally the covenants in the bond contract have coalesced the welfare of bondholders and stockholders
and shaped the recession equilibrium. Economic expansions (described above in Figure 3) are symmetric to the recession case but is omitted here to conserve space.

IV. Some Empirical Evidence

Is there any evidence that firms make investment decisions and financing decisions in the way predicted by the theory in section II and numerically illustrated in section III? To begin providing some evidence on this question we study the inter-temporal adjustments of the aggregate balance sheet for the U.S. nonfarm and nonfinancial corporate business sector over the period 1947-2002. This data is assembled by the Federal Reserve in their Balance Sheets for the U.S. Economy. We use the Wright (2004) formatting of this data for the nonfinancial corporate sector. From our perspective this sample has many desirable properties. To begin with, one of the important end uses of empirical studies of investment and financing decisions of firms is to obtain a deeper understanding of business cycles and the government policies designed to dampen them. Since the business cycle is an inter-temporal aggregate phenomenon, part of that understanding requires knowledge of the inter-temporal movements in aggregate investment and the financing of that investment. A second reason is that according to the model in Section II, corporate investment and business cycles are the result of an arbitrage between the market for real productive resources and the financial market. Whenever firms can buy or sell real assets in the product/factor market at economic book value prices different from the security market valuations of the claims against those assets, an investment driven business cycle occurs as was illustrated in the above numerical example. However, the most important advantage of the Federal Reserve data is that corporate assets and net worth are valued at replacement cost or economic book value rather than the historical cost accounting book value of the data sets used in many panel studies of investment and finance. The economic book value of assets can then be compared to the market valuations of the debt and equity securities used to finance them to get a true measure of the Q-ratio for the nonfinancial corporate sector. The actual data used in the regression tests below are described in the Appendix on Data Sources.
The regression/correlation analysis for the period 1947-2002 begins as it did in the numerical example with the asset adjustment or investment decisions of firms. In this model of divided decision-making among bondholders and stockholders, the investment decision represents the response of firms to changes in the market valuations of their equity shares. These changes in share valuations in turn reflect a shock-induced change in the re-pricing of risk by investors. According to the assignment rule in Section II the investment decisions of firms are made to conform to the risk aversion and risk perceptions of their stockholders. These asset adjustments from equations (12) and (13) change the operating income and operating risk of firms. To empirically proxy the asset adjustments we calculate the sum of the changes in the stock of inventories and plant and equipment for the nonfinancial corporate sector. This tangible investment variable is labeled ∆(Inv+P&E) and is our proxy for risky investments. Table 2 presents the estimated response of ∆(Inv+P&E) to six different lagged capital market variables from this sector that proxy for the capital market signal. The reason for the lag is that investment expenditures follow a time to build and before that a time to plan and order lag. The capital market signal influences the earlier planning stage of the capital accumulation process. Five of the six capital market variables are Q-ratios and changes in Q-ratios for both debt and equity securities as well as the entire corporate sector. For debt securities, the Q-ratio is labeled LTLiabQ, and in principle this variable equals the term R(b,ER)/R(b,RR) in equation (6). In the Federal Reserve balance sheets this variable is measured as the ratio of the market value of the sum of long-term bonds and mortgages to the book value of long-term bonds and mortgages of the nonfinancial corporate sector. The equity Q-ratio is labeled NWQ. In principle this variable is the term R(s, ER)/R(s, RR) in equation (11b). In the balance sheets this variable is measured as the ratio of the market value of net worth to the economic book value of net worth. The economic book value of net worth is measured as the difference between the replacement cost of assets.

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11 The Q-ratio relevant for investment is marginal Q. Hayashi (1982) has shown that marginal Q and average Q are equivalent when the firm is a price taker in product and factor markets and the production and installation functions are linear homogeneous. In Table 2 we proxy marginal Q with the level of measured Q, and the change in measured Q or ∆Q. To the best of our knowledge this is the first time Q has been split into a directly measured debt Q and an equity Q in a theoretical and empirical analysis of corporate investment.
and the book value of long-term liabilities. The fifth capital market variable used in this study is the real market value of the equity in the nonfinancial corporate sector, (SP). The sixth capital market variable used in the regressions is a combined debt and equity Q-ratio for the entire nonfinancial corporate sector and is simply labeled Q. Finally, to account for capital stock adjustment frictions a lagged value of the dependent variable is included as a regressor. The only other explanatory variable in the investment regressions is a dummy variable for the oil shock in the mid-1970s and is labeled DV-74 which takes on the value of 1.0 in 1974 and zero elsewhere.

The four parsimonious reduced form regressions 2.1-2.4 in Table 2 indicate that all of the estimated coefficients on the six lagged capital market variables are positive and statistically significant by conventional standards. This indicates that on average the balance sheet adjustments pass through zones I and III in Figure 2. The Breusch-Godfrey LM test indicates we cannot reject the hypothesis of no serial correlation in the regression residuals up to a lag of 2 years. The CUSUM plots of the recursive residuals (not shown here but available on request) all lie within the 5 percent upper and lower boundaries indicating that we cannot reject the hypothesis that the estimated coefficients are stable over the sample time period. These results are consistent with the predictions of the model in Section II. Changes in the capital market valuations of firms signal future changes in their risky investment expenditures as they implement the arbitrage between the financial market and the product market in order to generate the expected rates of return their investors now require. It is interesting to note that in both (2.1) and (2.2) the estimated coefficients on the two equity Q-ratios are on the order of 125 percent and 332 percent larger than the estimated coefficients on the two debt Q-ratios. One interpretation of this difference is that the equity signal is relatively more important than the debt signal in explaining the tangible investments of nonfinancial firms. This result is somewhat different from Philippon’s (2009) study where he found that a bond Q outperformed a traditional Q that combines debt and equity in explaining corporate investment. The importance of the equity signal is also confirmed in (2.3) where the explanatory variable is the lagged real market value of the equity in the nonfinancial corporate sector. Finally, in some unreported
regressions (but available on request) the addition of a lagged real cash flow variable (defined as the sum of depreciation charges and undistributed profits deflated by the consumer price index) had no material effect on the estimated coefficients or the statistical significance of the six capital market variables. The estimated coefficient on this lagged real cash flow variable was positive (in 2.1, 2.3, and 2.4) and negative in (2.2), but never statistically significant. Capital market imperfections were not detected in the aggregate data for nonfinancial corporations in our sample period.

The results in Table 2 are consistent with the view that various lagged Q-ratios and stock valuations trigger changes in the tangible asset decisions of nonfinancial enterprises. These arbitrage-induced asset adjustments can also cause business cycles. These asset adjustments through the return generating process in (12) and (13) also change the operating income and operating risk of firms. Any change in the operating risk of firms will change the investment quality and market valuations of corporate debt. Bondholders, of course, realize this. In the model described in Section II they negotiate an upfront contract with stockholders to offset any change in operating risk that accompanies the real investment decision with a financing decision that changes financial risk in the opposite direction. In the stylized example presented above the two part linked covenant achieved this objective. The question now is whether there is any statistical evidence that firms match their financing decisions to their investment decisions over time in the way predicted by the model in Section II.

------------------Insert Table 3 here------------------

Some evidence towards a beginning of an answer to this question is presented in Table 3. In the table the adjustment in long-term financial leverage is regressed on several measures of risky corporate investment. The financial leverage variable is measured as the change in the ratio of long-term liabilities to the replacement cost (i.e., economic book value) of total assets in the nonfinancial corporate sector, or, \( \Delta(LTLiab/A) \). In regressions (3.1) and (3.2) this measure of financial leverage is regressed on two measures of tangible investments: i) the change in the sum of the stock of inventories and plant and equipment used in Table 3, \( \Delta(Inv+P&E) \) and ii) the change in the stock of plant and equipment, \( \Delta(P&E) \). The lagged value of the dependent variable is also included as a regressor to measure the speed of
adjustment that reflects financial frictions. In addition to the dummy variable for the oil shock in the mid-1970s, dummy variables for the years 1986 and 1987 are included to account for the change in the tax regime that accompanied the Tax Reform Act of 1986. The Tax Reform Act of 1986 reduced personal income tax rates below the corporate rate thus providing firms with a tax inducement to finance their investments with debt. The results in these two regressions are similar. In both cases the estimated coefficient \( b_1 \) is negative and statistically significant as predicted by the theory. The Breusch-Godfrey LM test for serial correlation fails to reject at the 5 percent level the hypothesis of no serial correlation in the residuals up to a lag of 2 years. Again it is also the case that the CUSUM plots of the recursive residuals (not shown here but available on request) all lie within the 5 percent upper and lower boundaries indicating that we cannot reject the hypothesis that the estimated coefficients are stable over the sample period. An increase in tangible investment that increases the operating risk of nonfinancial firms by equation (13) is seen to be accompanied by a reduction in financial leverage and financial risk. The goal of rational financial contracting is to achieve this result. In regressions (3.3) and (3.5) of the table financial leverage is matched with two measures of investment in net working capital as described in the stylized numerical example above. In (3.3) net working capital is defined as the change in the difference between financial assets held by firms and their short-term liabilities. This difference is then deflated by the consumer price index and labeled \( \Delta(WC) \). In regression (3.5) nominal net working capital is scaled by total assets in the nonfinancial corporate sector. This ratio is then differenced to get \( \Delta(WC/A) \). In both regressions the estimated coefficient \( b_1 \) is predicted to be positive. This is because investment in net working capital represents a relatively safe investment strategy from the perspective of bondholders in that it is a relatively liquid form of collateral. As can be seen in the table the estimated coefficient \( b_1 \) on \( \Delta(WC) \) in (3.3) and \( \Delta(WC/A) \) in (3.5) are both positive and statistically significant. The Breusch-Godfrey test indicates an absence of serial correlation in the residuals up to a lag of 2 years. These two regressions indicate that a shift towards a relatively safe investment strategy that reduces operating risk is matched with a risky financial strategy that increases financial leverage and financial risk. Finally, regression (3.4) includes both \( \Delta(\text{Inv+P&E}) \) and \( \Delta(WC) \) as regressors. These results are more or less the same as before.
The estimated coefficient on $\Delta(\text{Inv+P&E})$ is negative and statistically significant, while the estimated coefficient on $\Delta(\text{WC})$ is positive (and roughly the same as in 3.3) and statistically significant. In summary this rough first pass at the evidence in Table 3 does not reject the hypothesis that firms in the nonfinancial corporate sector match their financial strategy to the risk characteristics of their investment strategy. Rational financial contracting as illustrated in the numerical example above would appear to be an important mechanism by which this matching is achieved.

V. Extensions and Directions for Future Research

The model in section II argues that the representative firm describing the supply side of the economy continually changes the size and composition of its balance sheet in response to changes in risk aversion and perceived risk of investors as reflected in the market valuations of its securities. It is these changes in market valuations (relative to current cost book valuations of assets) that create the arbitrage opportunities that initiate business cycles in this model. In response to a negative risk shock, this representative firm downsizes the level of its assets and reduces the proportion of risky assets generating high expected returns but also high risk. This downsizing and the shedding of risky assets causes a recession. At the same time the now safer downsized firm/economy shifts its financing towards debt thereby increasing financial leverage and financial risk. In response to a positive risk shock the representative firm does the opposite by expanding its assets and increasing the proportion of risky assets generating high expected returns but also high risk. This risky capital accumulation causes a cyclical expansion, which is then financed with equity. The question we now address is whether there are other ways this theory and its implications can be expressed.

A. Small and Medium Sized Young Firms versus Large Mature Firms

One way to do this is to split the single representative firm into two types of firms based on the operating risk their production-investment decisions generate as in Covas and Den Haan (2011), Crouzet and Mehrotra (2018), and Begenau and Salomao (2019). Towards this end, consider an economy where there are two types of firms offering their different investment and financing strategies for sale to
investors in the capital market. One group consists of established mature firms. These firms produce products that are well established in the market place and accordingly the subjective probability distributions describing the future returns on their investments are relatively well understood. Because of these attributes, they are able to finance their operations with long-term market debt and equity. Their name recognition in the product market allows them to issue their debt and equity securities in the national and international capital market. Much of their investments are in new and cost efficient capital goods embedding the latest technology. The second group are small and medium sized enterprises (SME’s) including new start-up companies. These firms are finance constrained. When they invest, some of it is in relatively cheap second hand capital goods purchased from mature firms.\textsuperscript{12} Their source of financing is mainly from equity (both common and convertible preferred stock) and bank loans. Equity investors would include family and friends, angel financiers, venture capitalists, and crowdfunding. Another source of equity financing is through PIPE’s, a private placement of equity at a price below current market. What about bank debt? In our model bank loans to SME’s are more like equity than the long-term market debt of mature firms. There are two reasons for this. The first is that banks have inside information on borrowers through observing their deposit balances over time (Fama, 1985). The second reason is that bank debt with tightly set covenants give the bank certain control rights over the future business plans of financed constrained SME borrowers (Dichev and Skinner, 2002) and Chava and Roberts (2008). These control rights have real effects on production and investment decisions. Bank loans that mature in and around the stockholder quarterly board of directors meeting, give bankers the opportunity to provide input for the future direction of these firms in terms of their investment and financing decisions. Directors acting for stockholders cannot ignore the views of bankers when in need of external financing. In this way banks have considerable control over the future business decisions of SME’s just like equity investors.

\textsuperscript{12} Rampini (2019) develops a model with these implications. For evidence on this see Eisfeldt and Rampini (2007).
So how do business cycles evolve in this set-up with these two types of firms? As before it all begins with an outside shock that changes investor’s risk aversion and/or their perception of risk. Suppose then a positive shock reduces the risk aversion of equity investors in banks and other firms driving up share prices and reducing the required rate of return of equity investors. Portfolio investors like banks and other institutional investors (including venture capitalists) shift their portfolio composition towards the risky loans and equity of SME’s. Existing small risky firms and new start-up firms now grow faster than mature firms and become a relatively larger part of the economy. Evidence consistent with this cyclical pattern of finance is provided in Erel et al. (2011). Some of this new finance is used to acquire second hand capital from mature firms who upgrade their capital stock during the cyclical expansion. This is consistent with the empirical observation that second hand capital acquisitions are more procyclical than investment in new capital (Eisfeldt and Rampini, 2006; Cao and Shi, 2016) reflecting a capital reallocation in the economy. How will this expansion in activity by SME’s be financed? It will be financed with equity and bank loans. A negative shock to risk aversion produces the opposite results. Banks now tighten their lending standards and SME’s become finance constrained. Finance now shifts away from SME’s towards well-established and relatively safe mature firms. Small firms find it harder to obtain financing to support their risky production-investment plans and many go bankrupt as banks reduce their lending to these borrowers. Much of their second hand capital remains idle or is scrapped. Mature firms scale back their production and investments but are better able to survive the ensuing recession than small firms because of their access to external financing from the capital market. The result is mature firms in established industries now become a bigger part of the economy. Moreover, during the recession phase of the business cycle as small firms disappear, long-term market debt becomes a relatively larger component of aggregate corporate financing compared to bank loans and equity.

What additional stylized facts emerge from this particular interpretation of the business cycle model? To begin with, the banks in our model behave much like the representative nonfinancial firm described above in Section II. A favorable outside shock that reduces bank shareholders required rates of return and increases bank share valuations is the market signal for banks to shift their portfolio towards the risky
loans of SME’s. This suggest that an upturn in general economic activity is the result of small firms getting the equity and bank finance they need to support their risky production-investment plans. Bank regulation in the form of the countercyclical buffer in the Basle Accord then forces these banks to finance this risky portfolio strategy increasingly with tier 1 equity. Evidence in support of these portfolio and financing adjustments for the U.S. and Euro area banks can be found in Krainer (2009)(2014). Bank loans like equity finance are procyclical and more or less match the procyclical growth in assets of small firms that generate an increase in future operating risk (as in equation 13) thereby sowing the seeds for a future recession. SME’s are an important engine of economic growth but also the source of economic volatility. Evidence consistent with this description of debt issuance of small firms and large firms is presented in Diamond (1991), Barclay and Smith (1995), Bolton and Freixas (2000), Denis and Mihov (2003), Erel et al.(2011), Custodio et al. (2013), and Becker and Ivashina (2014).

B. Labor Adjustments over the Business Cycle

The two categories of young versus mature firms discussed above suggests that this framework might also be useful in describing certain patterns of unemployment between young and mature workers in the labor market. Consider then a labor market consisting of young less risk averse apprentice workers and more risk averse and experienced mature workers. These two groups of workers resemble the mature firms (and bondholders) and SME’s (and stockholders) described above and in section II.

The question might then be raised as to whether there is a form of cyclical risk and return sharing among non-managerial workers that parallels the risk and return sharing among bondholders (mature firms) and stockholders (young firms) discussed in section II and V. Except for athletes in certain team sports, it is important to note that labor as a factor of production does not have a tradable market claim on the firm in the same sense as capital financed with marketable bonds and stocks. In addition, labor can differ in quality whereas the money provided to the firm by bondholders to buy productive resources is no different from money provided by stockholders. Yet the dichotomy of relatively more risk averse bondholders and less risk averse stockholders would seem to hold as a rough first approximation for labor inputs too. In this sense, it is possible to think of a set of utility enhancing institutional arrangements that
would produce a similar result in terms of risk and return sharing among groups of workers with different
aversions towards risk. To continue with this analogy suppose labor inputs are supplied by two distinct
groups of workers: i) mature and more efficient workers that are relatively more risk averse than, ii)
young inexperienced apprentice workers. Furthermore, suppose it is possible to a certain extent to trade
off a portion of wage income for job security much like it is possible to trade off expected return for risk
reduction in the form of priority for bondholders. In other words, the firm (or labor union) offers mature
workers the opportunity to buy a priority claim on the wage bill of the firm which they pay for in the form
of a lower wage relative to their marginal product. To develop this point further in the strongest possible
way, it is assumed that there is only one wage rate paid to both types of workers that reflects the marginal
product of a composite of the two types of workers. For the more senior workers this wage rate will be
below their marginal value product while for the young workers it will be above their marginal product.
In exchange for their lower wage relative to their marginal product the firm (and/or union) contracts to
give the more risk averse mature workers a certain degree of job security relative to the young and less
risk averse workers. What this means is that job losses will first fall on the young workers.\textsuperscript{13}

It might be asked why this particular institutional arrangement of seniority would ever arise in
efficient labor markets. Why, for example, couldn’t a more mature and productive worker save a portion
of his or her higher wage income based on their higher marginal value product and invest it in the capital
market thereby obtaining a sort of “homemade” security (that comes with seniority) to buffer future losses
in wage income caused by random shifts in the sectoral demand for labor? Part of the answer to this
question must be the cost of establishing separate labor markets for workers of different quality, and the
cost of transacting in the capital market. One example of the latter type of costs would be the hiring of an

\textsuperscript{13} For example, consider the unemployment rate of young workers (age 20-24) versus mature workers (age 45-54)
over the period 2002-2016 in the U.S. The unemployment rate for the young workers averaged 11 percent over the
period with a high of 15.5 percent in 2010 and a low of 8.2 percent in 2007. For the mature workers their
unemployment rate averaged 4.8 percent with a high of 7.7 percent in 2010 and a low of 3.1 percent in 2006. The
volatility of the unemployment rate as measured by the standard deviation was greater for the young workers than
the mature workers; namely, .0244 versus .015. Source: Labor Force Statistics from the Current Population Survey,
investment adviser to form a suitable portfolio for the mature worker. Financial services are expensive as shown by Philippon and Reshef (2009). More generally, if the cost of delivering security of income and consumption through seniority arrangements is less than the collective costs of obtaining that security in the capital market, then seniority arrangements will evolve in a natural way. When this is the case there will be fewer wage rates delivering labor income to workers than would prevail in the absence of seniority arrangements. Once obtaining this seniority these relatively risk averse mature workers have a priority claim to the wage bill of the firm relative to their “overpaid” but less risk averse young co-workers. The former are in effect bondholders while the latter are like stockholders. Thus in the risk and return sharing model of the labor market, young less risk averse workers choose an institutional arrangement involving erratic employment patterns in exchange for wage rates that are on average higher than their marginal product. In other words, their wage rate contains a risk premium (relative to the wage rate they should earn based on their marginal productivity) for the same reason that equity yields contain a risk premium relative to bond yields in the capital market. After all, these young and less risk averse workers know that they can—and often do—move in with their more risk averse parents when they are laid off in recessions. This might also be one reason why parents are relatively more risk averse than their children.

With this institutional arrangement in place (eg., one wage rate, two groups of workers with different risk aversions, and seniority) consider an initial equilibrium contract point like z in Figure 6. Now suppose there is an unexpected negative taste for risk shock that raises the required yield on equity shifting the ee schedule to e’e’ driving the market valuation of shares below economic book value. Market values of equity below economic book values are a market signal for managers to implement a safe and conservative production-investment strategy that takes the form of downsizing the firm. More particularly, the manager responds to this capital market signal by first laying-off the young less efficient and less risk averse workers and selling risky productive assets back to the product/factor market. This downsizing to become a safer firm causes a recession. As these young less risk averse workers are laid off the protective cushion they provide their more senior and risk averse co-workers is increasingly
diminished just as bondholder see their equity cushion diminished in a recession. The end result is that in the recession equilibrium at $z'$ in Figure 6 the more mature and risk averse workers have a larger share in the employment and labor income of a relatively safer representative firm just like bondholders. It is also the case as in the option-pricing model that bondholders (and mature workers) are the permanent investors (workers) in the firm. Again, it is important to remember that this particular analogy of workers as bondholders and stockholders is only a conjecture. Only subsequent research will determine whether it is a useful conjecture.

VI Summary and Conclusions

Implicit and explicit contracts are the glue that binds agents together in any cooperative venture be it a family of two or a modern complex firm with thousands of investors and workers. In our parsimonious model there are two groups of agents providing resources to the representative firm whose welfare must be coalesced together over time. One is a relatively more risk averse representative bondholder/mature worker and the second is a relatively less risk averse representative stockholder/young apprentice worker. For investors this coalescing takes the form of resolving a conflict of interest problem between the two differentially risk averse agents over the representative firm’s future investment and financing decisions. Bondholders prefer low risk business strategies because of their risk aversion and the payoff pattern of bonds; while stockholders prefer more risky decisions because of their risk aversion and the call option feature of their claim on the firm. Covenants in bond contracts are designed to replace trust and overcome these disagreements and allow the firm to move forward. Covenants are also the mechanism by which these agents confront the unknown risks of the future. They cannot know the future but they can contract for relative shares of whatever financing and investment returns materialize in the future.
In the model presented in Section II the managers of the representative firm formulate and implement two balance sheet decisions; an asset adjustment decision, and a financial adjustment decision. Changes in these decisions are a response to changes in the market valuations on stocks and bonds. Debt and equity investors throw out required rates of return in financial markets, and managers adjust their assets/labor force and financing over time to generate the equivalent expected rates of return on their debt and equity securities. Asset adjustment decisions, via the firm’s return generating function, generate the operating income and operating risk of the firm. These decisions can result in business cycles. Financing decisions produce financial risk by magnifying the underlying operating risk. The equilibrium condition for the representative firm/economy in Section II is when managers make operating decisions and financing decisions that deliver the expected yield that bondholders and stockholders require on their debt and equity investments in the firm. Observationally, this equilibrium condition is equivalent to the equality of the market value and economic book value of the debt and equity securities of the firm. At this equilibrium all arbitrage profits between: i) the bond and stock markets (in zones II and IV in Figures 3 and 4) and, ii) the financial market and product/factor market (in zones I and III of Figures 5 and 6) are exhausted. In this sense the business cycle is an equilibrium phenomena linked to changes in the risk perceptions and the taste for risk of investors. To counteract these changes the central bank might want to consider carrying out open

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14 In the model described in Section II financial leverage is predicted to be countercyclical. In contrast the traditional finance and macroeconomic literature emphasizes the existence of agency problems in the form of asymmetric information that creates a wedge between outside (debt) finance and inside (equity) finance. Following this path of analysis typically leads to the conclusion that financial leverage is pro-cyclical. This is because the value of net worth and the collateral value of assets on which outside borrowing is based in order to overcome these problems is pro-cyclical (see Bernanke and Gertler, 1989 and Kiyotaki and Moore, 1997). In addition these papers include short-term bank loans as debt whereas we count bank loans to be more like equity. This was discussed in section V. On the other hand the model described in Figures 5 and 6 of Section II is based on differences in risk aversion among bondholders and stockholders. This risk aversion changes over time in response to external shocks thereby changing investment and operating risk and operating risk of the representative firm via the return generating function in (12) and (13). A positive shock that reduces risk aversion and required rates of return increases the market valuations of bonds and stocks and causes investment and operating risk to increase as the economy expands. It is the increase in investment and operating risk in cyclical expansions that trigger the bond covenants that reduce financial leverage and financial risk. The result is that financial leverage is countercyclical. This prediction from the theory in Section II is not rejected by the data in Table 3.

15 One advantage of an arbitrage equilibrium is that it is in principle observable. Supply and demand schedules are not directly observable.
market operations in equity index funds to offset any sharp changes in the perceptions of and aversion towards risk of private investors.\textsuperscript{16}

In Section III a model bond contract was designed with real world covenants to guide the firm toward the no arbitrage equilibrium (point $z$) described in Section II. In developing this contract it was observed that with 2 investors, 2 business decisions, and 2 market equilibrium conditions, the no arbitrage equilibrium point $z$ in Figures 5 and 6 could be attained by assigning the 2 decisions to the 2 investors. According to the covenants in this contract, managers make investment decisions to conform to the risk perceptions and risk aversion of their stockholders as reflected in stock prices, and then make financing decisions to offset any change in the valuation of the bonds resulting from the investment decision.\textsuperscript{17} Alternatively, this could be broadly interpreted as having the stockholders manage the asset side of the balance sheet and bondholders manage the financing side. In practice this could be achieved by simply linking together two well-known covenants from the Commentaries in a two part contract. The first part of the contract would specify the amount of collateral (we arbitrarily chose net working capital) the firm would be required to maintain against its long-term funded debt. The second part of the contract would then determine the amount of cash the firm could distribute to its shareholders in the form of dividends and share repurchases, and this would be made to depend on the slack in the first part of the covenant. The advantage of linking the separate covenants in this way is that when the firm breaches the first part of the covenant with a speculative investment decision that initiates the expansion phase of the business cycle, a financing solution is automatically put in place in the second part of the covenant that guides the firm back towards compliance. A first pass at the regression/correlation evidence in section IV fails to reject the hypothesis that firms match financing decisions (and financial risk) to their investment

\textsuperscript{16} Central bank open market operations in equities have been proposed by Tobin and Brainard (1977), Fischer and Merton (1984), and Krainer (2003).

\textsuperscript{17} Our assignment rule is like the regulations contained in the various Basle Accords on capital requirements for banking firms. Bank managers acting in the interest of their stockholders (and themselves) make the portfolio decisions while regulators make the financing decisions to offset changes in the risk of the portfolio decisions. At the margin, an increase in portfolio risk in principle is matched with an increase in the equity leverage ratio. For the application of the model in Section II to the banking firm, see Krainer (2009).
decisions (and operating risk) over the business cycle as predicted by this contracting model. Finally, in
section V we suggested that the 2x2x2 framework of analysis developed in section II can be fruitfully
extended to include other twofold classifications in the study of certain aspects of the economy such as
the intertemporal balance sheet adjustments of SME’s versus large firms, and young versus mature
workers over the business cycle. Nevertheless the call for more theoretical and empirical work seems
particularly appropriate in order to further our understanding in how financial contracts coordinate the
investment and financing decisions of firms over the business cycle.
Appendix on Data Sources

NWQ = The ratio of the market value of net worth in the nonfinancial corporate sector to the economic book value of net worth. The economic book value of net worth is defined as the replacement cost of total assets minus the book value of long-term liabilities. From equation (11) $N(s)P(s)/K(s) = NWQ = 1.0$ when the expected rate of return on equity, $R(s, ER)$, equals the required rate of return, $R(s, RR)$. This was defined to be the equilibrium condition of the stock market. Obtained from Wright (2004). The data can be downloaded from his website www.econ.bbk.ac.uk/faculty/Wright.

LT Liab Q = The ratio of the market value of long-term liabilities (i.e., bonds and mortgages) to the book value of long-term liabilities. Recall from equation (6) that $N(b)P(b)/K(b) = LT Liab Q = 1.0$ when the expected rate of return on debt, $R(b, ER)$ equals the required rate of return on debt, $R(b, RR)$. This was defined to be the equilibrium condition in the bond market. Obtained from Wright.

SP = The real market value of equity in the nonfinancial corporate sector. Nominal market valuations were deflated by the consumer price index. Obtained from Wright.

Q = The ratio of the market value of nonfinancial corporate equity and net liabilities (total liabilities minus financial assets) to the replacement cost of BEA tangible assets. BEA and Federal Reserve measures of tangible assets differ in terms of their respective valuations of corporate real estate after 1989. After 1989 the Federal Reserve assumed that the real stock of land remained constant and the value of land was only adjusted for changes in real estate prices. The BEA measures land as a fixed proportion of total assets. In this study we use the BEA valuation. Obtained from Wright.


WC = Net working capital in the nonfinancial corporate sector. This variable is defined as the difference between financial assets and short-term liabilities and is deflated by the consumer price index. Data for financial assets and short-term liabilities available in Wright.

LT Liab/A = The ratio of long-term liabilities (the sum of bonds and mortgages) to the BEA economic book value of total assets in the nonfinancial corporate sector. Obtained from Wright.
REFERENCES


### Table 1

**Recession**

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<td>ST Assets</td>
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<td>Risky Assets</td>
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<tr>
<td>Equity $K(s)$</td>
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<td>Total $K$</td>
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<td>$\gamma$</td>
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<td>$X(b)$</td>
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<tr>
<td>$X(s)$</td>
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<tr>
<td>$R(b, ER)$</td>
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<td>0.459</td>
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<tr>
<td>$R(b, RR)$</td>
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<tr>
<td>$R(s, RR)$</td>
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<td>0.541</td>
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<tr>
<td>$R$</td>
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<td>$N(b)$</td>
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<td>4</td>
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<tr>
<td>$P(s)$</td>
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<td>$N(s)$</td>
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<tr>
<td>$P(s) N(s)$</td>
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<td>554.40</td>
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<tr>
<td>$P(b) N(b) + P(s) N(s)$</td>
<td>1000</td>
<td>954.40</td>
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<tr>
<td>$P(b) N(b) + P(s) N(s)$</td>
<td>300</td>
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Table 2

Nonfarm and Nonfinancial Corporate Tangible Investment
\[ \Delta(\text{Inv} + \text{P&E})_t = a_0 + a_1(\text{Capital Mkt. Variables})_{t-1} + a_2\Delta(\text{Inv} + \text{P&E})_{t-1} + a_3(\text{DV-74}) + U_t \]

1947 – 2002 (Annual)

<table>
<thead>
<tr>
<th>Regressions:</th>
<th>2.1</th>
<th>2.2</th>
<th>2.3</th>
<th>2.4</th>
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<td>Regressors</td>
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<tr>
<td>(\Delta\text{NWQ}_t)</td>
<td>3056.847</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(6.11/000)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Delta\text{LT Liab Q}_t)</td>
<td>1358.015</td>
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<td></td>
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<tr>
<td></td>
<td>(4.92/000)</td>
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<td></td>
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<tr>
<td>(\text{NWQ}_t)</td>
<td>3505.735</td>
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<tr>
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<td>(3.45/001)</td>
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<td>(\Delta\text{Q}_t)</td>
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<td>(5.25/000)</td>
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<tr>
<td>(\Delta(\text{Inv} + \text{P&amp;E})_t)</td>
<td>.640</td>
<td>.530</td>
<td>.378</td>
<td>.510</td>
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<td></td>
<td>(6.45/00)</td>
<td>(5.90/00)</td>
<td>(2.99/01)</td>
<td>(4.39/00)</td>
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<tr>
<td>(R^2)</td>
<td>.50</td>
<td>.42</td>
<td>.52</td>
<td>.53</td>
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<td>Breusch-Godfrey Test</td>
<td>.368/.69</td>
<td>2.11/1.3</td>
<td>2.62/0.8</td>
<td>1.48/.24</td>
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The table reports the estimated slope coefficients on \(a_1\) and \(a_2\) in four tangible investment regressions for the nonfarm and nonfinancial corporate sector. The Newey-West corrected t-scores and p-values are reported beneath the estimated coefficients.

\[ R^2 = \] Adjusted coefficient of determination.

The Breusch-Godfrey LM test is a test for serial correlation in the regression residuals up to a pre-specified lag. The first number reported is the F-statistic and the second is the P-value. The null is that there is no serial correlation in the residuals up to a lag of 2 years. Low probabilities (e.g., Pr<0.01) indicate a rejection of the null.
Table 3

Financial Leverage Adjustments in the Nonfarm and Nonfinancial Corporate Sector

\[ \Delta(\text{LT Liab}/A)_t = b_0 + b_1(\text{Corporate Investments})_t + b_2\Delta(\text{LT Liab}/A)_{t-1} + b_3(\text{DV} - 74) + b_4(\text{DV} - 86) + b_5(\text{DV} - 87) + V_t \]

1947—2002 (Annual)

<table>
<thead>
<tr>
<th>Regressions:</th>
<th>3.1</th>
<th>3.2</th>
<th>3.3</th>
<th>3.4</th>
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</thead>
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<tr>
<td>Regressors</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta(\text{Inv} + \text{P&amp;E}) )</td>
<td>-.0139(^a)</td>
<td>-.0177(^a)</td>
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<tr>
<td></td>
<td>(-2.30/.025)</td>
<td>(-3.85/.000)</td>
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</tr>
<tr>
<td>( \Delta(\text{P&amp;E}) )</td>
<td></td>
<td>-.0163(^a)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(-2.01/.050)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta(\text{WC}) )</td>
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<td></td>
<td>.0170(^a)</td>
<td>.0198(^a)</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(3.65/.001)</td>
<td>(5.59/.000)</td>
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</tr>
<tr>
<td>( \Delta(\text{WC}/A) )</td>
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<td></td>
<td></td>
<td></td>
<td>.2973</td>
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<tr>
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<td></td>
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<td></td>
<td>(4.27/.000)</td>
</tr>
<tr>
<td>( \Delta(\text{LT Liab}/A)_{t-1} )</td>
<td>.375</td>
<td>.368</td>
<td>.251</td>
<td>.265</td>
<td>.203</td>
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<tr>
<td></td>
<td>(2.71/.01)</td>
<td>(2.68/.01)</td>
<td>(2.19/.03)</td>
<td>(2.58/.01)</td>
<td>(1.70/.10)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>.41</td>
<td>.39</td>
<td>.46</td>
<td>.56</td>
<td>.59</td>
</tr>
<tr>
<td>Breusch-Godfrey Test</td>
<td>1.97/.15</td>
<td>1.76/.18</td>
<td>2.71/.08</td>
<td>2.70/.08</td>
<td>2.40/.10</td>
</tr>
</tbody>
</table>

\(^a\) = The estimated coefficient and the standard error of the coefficient have been multiplied by 100.

The table reports the estimated slope coefficients on \( b_1 \) and \( b_2 \) in five financial leverage regression for the nonfarm and nonfinancial corporate sector. The Newey-West corrected t-scores and p-values are reported beneath the estimated coefficients.

\( R^2 \) = Adjusted coefficient of determination.

The Breusch-Godfrey LM test is a test for serial correlation in the regression residuals up to a pre-specified lag. The first number is the F-statistic and the second is the P-value. The null is that there is no serial in the residuals up to lag 2. Low probabilities (e.g., Pr<0.01) indicate a rejection of the null.
Internet Appendix

The objective of this appendix is to motivate the transition of an all equity start-up firm to a debt and levered equity firm based on differences in risk aversion. Towards this end suppose a risk neutral entrepreneur starts a representative firm/economy. This is described in Figure A1 with the well-known Fisher/Hirshleifer diagram. The horizontal axis represents current resources OP that can be used for current consumption $C_0$ and current real investment $K_0$ that yields an expected future income, $\bar{X}$. This future income is presented on the vertical axis. The curve PO represents a diminishing returns real investment opportunity describing how current resources in the form of real investment can be converted into expected future resources. The indifference curve IC describes the preferences of the entrepreneur for current consumption and future expected income. The well-known tangency solution that maximizes utility at M describes the equilibrium for the risk neutral entrepreneur with $C_0$ of current consumption and $K_0$ of current investment generating $\bar{X}$ units of expected future income. The box in the right side of the diagram represents the all equity-financed firm owned by the entrepreneur.

Figure A1

Consumption and Investment of a risk neutral investor

![Diagram of consumption and investment of a risk neutral investor]
Now suppose the risk neutral entrepreneur retires and bequeaths the all equity firm/economy equally to his/her differentially risk averse children, investors B and S. The firm/economy can now be described in terms of an Edgeworth-Bowley box diagram as in Figure A2.

**Figure A2**

*Input and Expected Output Sharing in an All Equity Economy*

The mean expected returns and variance of returns for this firm/economy are respectively $\bar{X} = 50$ and $\sigma^2(X) = 833.33$. With unlevered equity investor B and S each have expected returns of 25 and variance of returns of 208.33.

Next we suppose B and S have the following specific decreasing absolute risk-aversion utility functions.

$U(X)_b = \ln (1+X)$

$U(X)_s = .7X - e^{-0.02X}$
Finally, we assume the promised payment $X(b)$ on any bond that might be issued by the firm is 25, although in this numerical example $24 < X(b) < 29$ would also work.

To see whether an all equity capital structure is preferred to a debt and levered equity capital structure we compute the expected utility for each investor for both capital structures. For the all equity capital structure, we get the following expected utility for investors $B$ and $S$ respectively:

$$E[U(X)_b] = \int_{0}^{50} \ln(1 + x) f(x)dx$$

$$= .02 \left[ (1 + X) \ln(1 + X) - (1 + X) \right]_{0}^{50} = 3.010462$$

and

$$E[U(X)_s] = \int_{0}^{50} (.7X - e^{-0.02X})f(X)dx$$

$$= .02 \left[ .7 \frac{x^2}{2} + \frac{1}{0.02} e^{-0.02X} \right]_{0}^{50} = 16.86788$$

For the debt and levered equity capital structure, we get for investor $B$:

$$E[U(X)_b] = \int_{0}^{25} \ln(1 + x) f(x)dx + \int_{25}^{100} \ln(26) f(x)dx$$

$$= .01 \left[ (1 + X) \ln(1 + X) - (1 + X) \right]_{0}^{25} = 3.0406775$$

For investor $S$:

$$E[U(X)_s] = \int_{0}^{25} [.7(0) - e^{-0.02(0)}] f(x)dx + \int_{25}^{100} [.7(X - 25) - e^{-0.02(X-25)}] f(x)dx$$

$$= (-.25) + (.01)[ .7 \frac{x^2}{2} - 17.5X + \frac{1.64872}{0.02} e^{-0.02X}]_{25}^{100} = 19.0491$$

As can be seen both investors $B$ and $S$ prefer the debt and levered equity capital structure to the all equity capital structure. Actually, it can be shown that investor $B$ would obtain a still higher expected utility of 3.05 if he/she could invest in 80 percent of the bonds and 20 percent of the levered equity. However, this would require investor $S$ to hold 20 percent of the debt and 80 percent of the levered equity, which would reduce $S$’s expected utility to 18.1794. Investor $S$ who can hire and fire the managers would instruct them to reject this financial structure. The firm should then issue bonds to investor $B$ and levered stock to investor $S$ in exchange for their unlevered stock. When $B$ owns the bonds it can be shown for this numerical example their expected income is $\bar{X}(b)=21.875$, and their variance of income is $\sigma^2= 42.08$. For investor $S$ we get $\bar{X}(s)= 28.125$ and variance of income of $\sigma^2= 615.231$. With the new financial structure investor $B$ is better off in terms of risk reduction while investor $S$ is better off in terms of expected income. The firm/economy’s capital structure where $B$ owns all the debt and $S$ the levered equity is described in Figure A3, which is Figure 1 in the text.
It is curious that the Edgeworth-Bowley box diagram in Figures 1, A2, and A3 have played no role in the development of ideas in finance. This is unfortunate since it offers a simple illustration of the Modigliani-Miller (MM, 1958) financial structure proposition in a tax-free environment. This famous proposition is contained in their equation 4 and simply states that the cost of capital for a firm is (in our notation): $\bar{X}/K = R$ for any firm $j$ in risk class $k$. It is important to note that in MM’s proof the proposition only held for firms in the same risk class where the probability distributions across firms and their real investments are constant. Once the firm’s capital stock $K$ and expected returns $\bar{X}$ are fixed—as in the Edgeworth-Bowley diagrams—the cost of capital is completely independent of its capital structure defined as $K(b)/K(s)$. What changes when a firm changes $K(b)/K(s)$? To answer this MM assume (controversially) that the yield on debt $R(b)$ is a constant and independent of $K(b)/K(s)$. This case is presented in Figure A4. In Figure A4 the straight line emanating from the southwest corner of the box through $Z$ and towards the right vertical axis represents the constant $R(b)$. Along that straight line changes in $K(b)$ and hence $K(b)/K(s)$ plot various $Z$ points each one associated with a constant $R(b)$ but with $R(s)$ increasing linearly with $K(b)/K(s)$. In MM this is described by their equation 8 which is:

$$R(e) = R + [R - R(b)]K(b)/K(s)$$

In Figure A4 a straight line emanating from the northeast corner of the box through $Z$ towards the bottom horizontal axis describes this relationship between $R(s)$ and $K(b)/K(s)$. As $K(b)$ and...
K(b)/K(s) increase, the slope of the equity line with the embedded R(s) increases. In other words, the seemingly cheaper debt is exactly offset by an increase in the yield on levered equity resulting in the same constant cost of capital \( R = \bar{X}/K \). However, as we will see below their financial structure proposition will not hold over the intertemporal business cycle for reasons they well recognized; namely, the risk class (as described by a probability distribution) of the representative firm changes in different stages of the business cycle.

Figure A4

Input and Expected Output Sharing in a Debt and Equity Economy

\[
\begin{array}{c}
\overline{X} \\
\overline{X}(B)
\end{array}
\]