

Risk Adjustment and the Temporal Resolution of Uncertainty: Evidence from Options Markets

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Abstract

Risk-neutral probabilities, observable from option prices, combine objective probabilities and risk adjustments across economic states. We consider a recursive-utility framework to separately identify objective probabilities and risk adjustments using only observed market prices. We find that a preference for early resolution of uncertainty plays a key role in generating sizeable risk premia to explain the cross-section of risk-neutral and objective probabilities in the data. Failure to incorporate a preference for the timing of the resolution of uncertainty (e.g., expected utility models) can significantly overstate the implied probability of, and understate risk compensations for, adverse economic states.

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Introduction

By the Fundamental Theorem of Asset Pricing, the market price of any asset is given by the present value of its payoffs computed under the risk-neutral probability measure (Dybvig and Ross, 1987). These risk-neutral probabilities embed both the objective probabilities of economic states and the risk compensations that investors require for reaching these states. Empirical evidence suggests that risk-neutral probabilities vary substantially and increase sharply for adverse economic states, which implies substantial cross-sectional variation in either objective probabilities, risk compensations, or both. Identifying and quantifying the relative importance of these two channels for asset prices is one of the central open questions in financial economics, which has significant implications for our understanding of the economic risks in financial markets. In this paper, we develop and implement a recursive utility-based framework to separately identify the objective probabilities and risk adjustments using only financial market data, primarily equity index options. We show that the preference for early resolution of uncertainty plays a key role in generating sizeable risk adjustments which can quantitatively explain the cross-section of both the risk-neutral and objective probabilities in the data. Models which do not incorporate an effective risk premium channel (e.g., expected utility) significantly overestimate the implied probabilities of bad economic states and underestimate the role of risk adjustments. This difference, we show, has important implications for our understanding and interpretation of asset markets.

The theoretical framework of our paper builds on a recent literature which shows that risk-neutral probabilities, observable from option prices, have substantial information for both objective probabilities and risk adjustments, and under certain conditions can fully identify the two in the data. Hansen and Scheinkman (2009) establish a general framework to analyze the risk and return relationship, and use an operator approach to theoretically characterize the risk adjustments in Markov state environments. In a more specific application, Ross (2013) shows that physical probabilities and risk adjustments can be jointly recovered from risk-neutral probabilities when agents' preferences belong to a class of state-independent expected utility models.¹ In this paper, we consider an empirical identification of physical probabilities and risk adjustments in a more general environment which incorporates the Kreps and Porteus (1978) recursive preferences of Epstein and Zin (1989) and Weil (1989). Indeed, when agents' preferences are characterized by state-independent expected utility, the stochastic discount factor only depends on the realized state next period. Then, as shown in Ross (2013), the requirement that implied physical probabilities sum to one provides enough identifying restrictions to separate risk-neutral probabilities into

¹The Appendix in Ross (2013) considers a very special case of recursive preferences where the wealth portfolio is equal to the market and does not pay any dividends. Such a specification can not be supported in equilibrium; further, the full analysis of its implications has not been entertained.

physical probabilities and risk adjustments. However, under recursive preferences, agents also care about current economic conditions which affect the future evolution of wealth. We take this into account and develop a recursive utility-based framework to recover physical probabilities and risk adjustments from risk-neutral probabilities, given measurements of the wealth-to-consumption ratio and a preference parameter which captures the preference for the timing of the resolution of uncertainty. By comparing model-implied physical probabilities of the state dynamics to the observed probabilities in the data, we can identify the magnitude of this preference parameter, and thus fully characterize the risk adjustments in the data.

We first use a calibrated economic model to illustrate the role of recursive utility for identifying physical probabilities and risk adjustments. In particular, we consider a Mehra and Prescott (1985) economy with recursive preferences, as in Weil (1989), calibrated to match the dynamics of consumption, dividends, the observed equity premium, and the risk-free rate. We solve our economic model for the equilibrium risk-neutral probabilities and asset prices, and use them to recover physical probabilities and risk adjustments under alternative assumptions for the preference for the timing of the resolution of uncertainty. We show that unless the econometric framework correctly accounts for a preference for early resolution of uncertainty, we obtain biased estimates of physical probabilities, risk adjustments, and in turn biased estimates of average stock returns.

Intuitively, when the magnitude of the preference for early resolution of uncertainty is not fully accounted for, the recovery framework trades-off higher probabilities of bad economic states for lower risk compensations in these states. In the limiting case of expected utility, it is effectively a manifestation of the failure of the expected utility model to generate sizeable risk premia given smooth consumption dynamics. For example, while the unconditional probability of the lowest consumption growth state is 25% in the benchmark calibration, under the assumption of expected utility the implied physical probability is over 60%. As a consequence of these excessively large probabilities of bad states, the implied population averages of economic variables are significantly biased downward relative to their equilibrium values. For equity returns, the implied average return is less than 1% under expected utility, relative to the 9% calibrated value in the economic model. Hence, the quantitative implications of ignoring the preference for early resolution of uncertainty can be quite significant.

Next, we implement our framework using S&P 500 options data. Specifically, we first identify three aggregate economic states based on the past three-month returns on the S&P 500 index, with the worst state corresponding to the bottom 25% of the unconditional distribution of market returns and the best state to the top 50% of market returns. In the

data, the worst state reflects adverse aggregate economic conditions, with low market returns (-35.51%), low aggregate real consumption growth (0.29%), and a high VIX measure of implied uncertainty (30.29%), relative to high average annualized market returns (22.63%), high real consumption growth (1.79%) and low VIX (20.22%) in the best state. We use S&P 500 option prices to estimate the conditional risk-neutral probabilities of future stock prices in each state following the approach of Figlewski (2008); in particular, the left and right tail of the risk-neutral probability measure are fitted using the Generalized Extreme Value (GEV) distribution. Finally, consistent with the economic model, log wealth-to-consumption ratios are assumed to be proportional to log price-dividend ratios.

Using the observed asset price data, we recover physical probabilities and risk adjustments for different preference parameters. Our empirical findings are in line with the evidence from the economic model. We find that the implied probability of a bad state is significantly overstated without a sufficient preference for early resolution of uncertainty. Indeed, when the preferences collapse to expected utility, the implied probability of bad states is about 60%, compared to its fixed value of 25%. This has direct implications for the estimated risk compensation across states: under expected utility, the magnitude of the stochastic discount factor going to the bad state is more than two times smaller than going to the good state, while the opposite is the case under early resolution of uncertainty. The optimal preference structure which minimizes the distance between the recovered and actual probabilities of the states in the data is very close to the one entertained in our calibrated model, and for risk aversion greater than one, suggests that the inter-temporal elasticity of substitution parameter is above one and that the agent has a strong preference for early resolution of uncertainty. We verify that these key implications are robust to alternative identification of economic states in the data.

Overall, our paper makes a direct contribution to the debate about the relative importance of objective probabilities versus risk adjustments to explain asset prices. Indeed, as shown in Mehra and Prescott (1985), standard expected-utility models fail to capture asset price data because consumption growth is too smooth in the data and the implied risk premia are too low. One approach in the literature is to introduce more variation in the objective consumption dynamics through rare macroeconomic disasters, either perceived or actual, as in Rietz (1988) and Barro (2006). Another approach is to enhance the amount of model-implied risk compensation through an alternative preference structure, e.g. the recursive utility of Epstein and Zin (1989). In our paper, we show that the risk adjustments which arise under early resolution play a key role in explaining the variation in risk-neutral probabilities in the data, relative to the objective probabilities. These findings are consistent with Backus, Chernov, and Martin (2011), who show that it is challenging to explain observed option prices using extreme macroeconomic events under the objective measure.

Our findings are quantitatively consistent with the long-run risks literature, which underscores the importance of a high inter-temporal elasticity of substitution and a preference for early resolution of uncertainty to explain the key features of asset markets. Bhamra, Kuehn, and Strebulaev (2010) show that such a framework can generate significant risk premia to account for the observed prices of corporate bonds given low probabilities of defaults in the actual data. In a similar vein, Shaliastovich (2012), Drechsler (2013), and Eraker and Shaliastovich (2008) consider the equilibrium implications of the recursive utility structure for option markets and show that jumps in economic uncertainty can plausibly explain the prices of out-of-the-money puts. More broadly, Bansal, Dittmar, and Lundblad (2005), Hansen, Heaton, and Li (2008), Bansal, Kiku, Shaliastovich, and Yaron (2013) provide cross-sectional evidence for the economic channels in the long-run risks model, and Bansal, Kiku, and Yaron (2007) and Bansal, Kiku, and Yaron (2011), Hasseltoft (2012) consider the empirical evaluation of the model and find support for it in the data.² In a related equilibrium literature, Liu, Pan, and Wang (2005) and Gabaix (2012) consider the role of rare jumps in explaining out-of-the money option prices. Bekaert and Engstrom (2009) introduce an alternative non-Gaussian bad environment - good environment specification of consumption dynamics and show it can realistically capture risk-neutral distributions of equity returns in the data.

In our paper, we develop and implement a market-based approach to separately estimate physical probabilities and risk adjustments from the risk-neutral probabilities. Traditionally, the main approach in the literature to identify economic transitions and risk adjustments is to specify parametric models for the physical and risk-neutral dynamics (equivalently, for the physical measure and the risk adjustments); see e.g. Pan (2002), Eraker, Johannes, and Polson (2003), and Broadie, Chernov, and Johannes (2007). Andersen, Fusari, and Todorov (2012) develop a parametric estimation approach for state recovery based on a panel of options prices. Alternatively, Jackwerth (2000), Ait-Sahalia and Lo (2000), and Bliss and Panigirtzoglou (2004) use non-parametric methods to estimate the physical and risk-neutral distributions from the data to derive implications for equilibrium risk adjustments and preference parameters of the representative agent. Jurek (2009) uses a similar method involving transformations of the payoff function adapted from Bakshi, Kapadia, and Madan (2003) to obtain model-free estimates of the moments of the risk-neutral distribution from option prices. Carr and Yu (2012) develop the recovery framework in a bounded diffusion context based on restricting the dynamics of the numeraire portfolio.

²There is a large literature on the magnitude of the IES. Hansen and Singleton (1982), Attanasio and Weber (1989), Guvenen (2001) and Vissing-Jorgensen and Attanasio (2003) estimate the IES to be in excess of 1. Hall (1988) and Campbell (1999) estimate the IES to be well below one. Bansal and Yaron (2004) argue that the low IES estimates of Hall and Campbell are based on a model without time varying volatility.

Our paper proceeds as follows. In Section 1 we describe our theoretical framework. In Section 2 we set up our model economy, and use model calibrations to highlight the biases that arise without properly considering the preference for early resolution of uncertainty. Section 3 is devoted to our empirical analysis, while various robustness checks are carried out in Section 4. Section 5 concludes the paper.

1 Theoretical Framework

1.1 Setup of the Economy

We consider a Markovian regime-switching economy with N discrete states. As is typical in the literature, we specify our economy to be stationary in growth rates, in line with classic models such as Mehra and Prescott (1985) and Weil (1989) where consumption growth rates and dividend growth rates are stationary and follow a regime-switching process. Under the stationarity of growth rates, the levels of consumption and asset prices follow random walk processes and are unbounded, which is economically appealing; Dubynskiy and Goldstein (2013) discuss further drawbacks for recovery methods under stationary consumption levels.

Let $m_{i,j}$ denote the stochastic discount factor (SDF), which in standard structural models corresponds to the intertemporal marginal rate of substitution of the representative agent between state i today and state j next period. According to no-arbitrage pricing conditions, the value of an asset is given by the expectation of its payoff times the SDF. In particular, let $q_{i,j}$ denote the price of an Arrow-Debreu security, which is a state-contingent claim in state i that pays one unit of consumption in state j next period. From the pricing equation, the Arrow-Debreu price is given by the product of the SDF and the physical transition probability $p_{i,j}$:

$$q_{i,j} = m_{i,j} p_{i,j}. \tag{1.1}$$

By construction, the sum of the Arrow-Debreu prices given the current state is equal to the price of a one-period risk-free bond in this state, and the Arrow-Debreu price scaled by the price of a risk-free bond, $q_{i,j} / \sum_j q_{i,j}$, corresponds to the risk-neutral probability of going from state i today to state j next period. Following Breeden and Litzenberger (1978), we can estimate the risk-neutral probabilities in a model-free manner from the cross-section of options prices (see Section 3.2). Hence, the Arrow-Debreu prices can be directly identified using market data on option and bond prices alone.

Arrow-Debreu prices incorporate information on both physical probabilities and risk adjustments, as shown in (1.1). To identify the physical probabilities and risk adjustments separately, a standard approach in the literature is to use separate parametric or non-parametric models for the physical and risk-neutral dynamics (equivalently, physical dynamics and the SDF), which are then estimated in the data. Hansen and Scheinkman (2009) establish an alternative framework and use operator approach to theoretically characterize the risk adjustments in Markov state environments. Specifically, as shown in Ross (2013), under restrictions on the class of underlying preferences in the economy, the physical transition probabilities and the SDF can be recovered directly from the Arrow-Debreu prices. We highlight the main steps of this approach below, and then extend the market-based identification framework to the case of recursive utility.

1.2 Expected Utility Structure

Consider a specification where the representative agent has state- and time-independent expected utility. In this case, the marginal utility between consumption in states i and j depends only on the state the economy transitions to and not on the current state. For example, in the classic model of Mehra and Prescott (1985), the agent has expected power utility over future consumption. In this case, the SDF is independent of the current state, and is given by:

$$m_{i,j} = m_j = \delta \lambda_j^{-\gamma}, \tag{1.2}$$

where δ is the agent's time preference parameter, λ_j is the consumption growth rate in state j , and γ is the coefficient of relative risk aversion. With power utility, the SDF is directly related to the endowment dynamics and the preference parameters of the agent. Our subsequent analysis is more general and is not limited to power utility preferences. In fact, it is valid under any specification of time- and state-independent expected utility, and only relies on the identification assumption that the SDF does not depend on the current state.

Let Q denote the matrix of Arrow-Debreu prices, and P the matrix of physical transition probabilities. Let M be the diagonal matrix with the stochastic discount factors m_j on the diagonal. Then, we can rewrite (1.1) in matrix form:

$$Q = PM. \tag{1.3}$$

or more explicitly,

$$\begin{bmatrix} q_{1,1} & \cdots & q_{1,N} \\ \vdots & \ddots & \dots \\ q_{N,1} & \cdots & q_{N,N} \end{bmatrix} = \begin{bmatrix} m_1 p_{1,1} & \cdots & m_N p_{1,N} \\ \vdots & \ddots & \dots \\ m_1 p_{N,1} & \cdots & m_N p_{N,N} \end{bmatrix}. \quad (1.4)$$

With N discrete states, this is a system of N^2 equations in $N^2 + N$ unknowns. As pointed out in Ross (2013), to identify the physical probabilities and the SDF separately one can use the further restriction that conditional physical probabilities must sum to 1:

$$P \mathbf{1} = \mathbf{1} \quad (1.5)$$

This gives us N additional restrictions, which allows us to identify uniquely both the physical transition probabilities P and the marginal utilities M . Indeed, using the restrictions for the physical probability matrix in (1.3) and (1.5), we obtain:

$$P \mathbf{1} = Q M^{-1} \mathbf{1} = \mathbf{1} \implies M^{-1} \mathbf{1} = Q^{-1} \mathbf{1}. \quad (1.6)$$

The last condition completely characterizes M since it is a diagonal matrix:

$$M = [\text{diag}(Q^{-1} \mathbf{1})]^{-1}. \quad (1.7)$$

We can further recover the implied physical probability transition matrix:

$$P = Q \text{diag}(Q^{-1} \mathbf{1}). \quad (1.8)$$

By construction, each row of P sums to one, so subject to the requirement that the recovered P has all positive elements, it represents a valid matrix of conditional probabilities. The implied physical probabilities and the SDF satisfy the pricing equation (1.3), and further, by construction, this decomposition is unique.

Notably, in the analysis above, the physical probabilities and risk adjustments are identified using market data from Arrow-Debreu prices alone, and do not rely on the exact functional form of the SDF, the specification of the endowment process or the agent's preference parameters. Hence, the framework can be implemented empirically using only market data on options and bonds, which are arguably better measured relative to macroeconomic variables. In the next section we consider a recursive utility setup which no longer admits a state-independent SDF. We show that we can maintain a convenient market-based approach for extracting the physical probabilities and the SDF separately from the data, given

measurements of the wealth-to-consumption ratio and the preference parameter capturing the preference for the timing of resolution of uncertainty.

1.3 Recursive Preferences Structure

For the Kreps and Porteus (1978) recursive utility of Epstein and Zin (1989) and Weil (1989), the life-time utility of the agent V_t satisfies,

$$V_t = \left[(1 - \delta)C_t^{1-\frac{1}{\psi}} + \delta \left(\mathbb{E}_t \left[V_{t+1}^{1-\gamma} \right] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}} \quad (1.9)$$

where δ is the time discount factor, $\gamma \geq 0$ is the risk aversion parameter, and $\psi \geq 0$ is the intertemporal elasticity of substitution (IES). For ease of notation, the parameter θ is defined as $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$. Note that when $\theta = 1$, that is, $\gamma = 1/\psi$, the above recursive preferences collapse to the standard case of expected utility. As is pointed out in Epstein and Zin (1989), in this case the agent is indifferent to the timing of the resolution of uncertainty of the consumption path. When risk aversion exceeds the reciprocal of IES, the agent prefers early resolution of uncertainty of consumption path, otherwise, the agent has a preference for late resolution of uncertainty.

Note that when the risk aversion coefficient γ is bigger than one, the sign and the magnitude of θ have a direct economic implication for the preference structure of the agent. In this case, $\theta < 0$ implies that the inter-temporal elasticity of substitution is bigger than one, and further, the agent has a preference for early resolution of uncertainty ($\psi < 1/\gamma$). The IES goes below one when θ is positive, and when $\theta > 1$ the agent has a preference for late resolution of uncertainty; the limiting case of $\theta = 1$ nests standard expected utility. The relative values of the IES and the preference for the temporal resolution of uncertainty embedded in θ have significant implications for the equilibrium choices of the agent and for asset prices. For example, the long-run risks literature argues for the inter-temporal elasticity of substitution above one and for a preference for early resolution of uncertainty to explain the key features of asset markets, which suggests θ is negative (see Bansal and Yaron (2004)). An alternative interpretation of θ arises in the robust control and model uncertainty literature (see e.g. Hansen and Sargent (2010)), where the parameter $\theta < 0$ captures the agent's aversion to model mis-specification.

As shown in Epstein and Zin (1989), the real stochastic discount factor implied by these preferences is given by

$$m_{i,j} = \delta^\theta \lambda_j^{-\frac{\theta}{\psi}} R_{c,i,j}^{\theta-1} \quad (1.10)$$

where λ_j is the growth rate of aggregate consumption and $R_{c,i,j}$ is the return on the asset which delivers aggregate consumption as its dividends each time period (the wealth portfolio). This return is different from the observed return on the market portfolio as the levels of market dividends and consumption are not equal: aggregate consumption is much larger than aggregate dividends. Let us decompose the consumption return into its cash flow growth rate, λ_j , and the change in price-dividend ratio on the wealth portfolio PC between states i and j :

$$R_{c,i,j} = \lambda_j \frac{PC_j + 1}{PC_i}. \quad (1.11)$$

Substitute the return decomposition above into the recursive utility SDF in (1.10) to obtain:

$$m_{i,j} = \left[\delta^\theta \lambda_j^{-\gamma} (PC_j + 1)^{\theta-1} \right] PC_i^{1-\theta}. \quad (1.12)$$

Therefore, in the case of the recursive preferences, the SDF depends on both current and future economic states. Unlike the power utility case where agents care just about the next-period consumption shock, with recursive preferences they are concerned about the endogenous dynamics of their wealth, so that the economic variables which affect their wealth now determine the marginal rates of substitution between the periods. When $\theta = 1$, the preferences collapse to standard expected utility in (1.2), and the SDF depends only on the next-period consumption growth rate λ_j .

Note that the Epstein-Zin SDF can be decomposed multiplicatively into a component which depends only on the next-period state, \tilde{m}_j , and the component which depends only on the current state through $PC_i^{1-\theta}$:

$$m_{i,j} = \tilde{m}_j PC_i^{1-\theta}. \quad (1.13)$$

The SDF component which involves the next-period state, \tilde{m}_j , in general depends on the endowment dynamics and all the preference parameters. However, if we can characterize the current-state component $PC_i^{1-\theta}$, we can extend the recovery approach and identify the physical probabilities and the SDF directly using market data alone. Indeed, let us modify the matrix equation for the Arrow-Debreu prices (1.3) in the following way:

$$Q = PC(\theta)P\tilde{M}, \quad (1.14)$$

where $PC(\theta)$ and \tilde{M} are the diagonal matrices of $PC_i^{1-\theta}$ and \tilde{m}_i , respectively. Then, from the condition that physical probabilities sum to one, we obtain:

$$P\mathbf{1} = PC(\theta)^{-1}Q\tilde{M}^{-1}\mathbf{1} = \mathbf{1} \implies \tilde{M}^{-1}\mathbf{1} = Q^{-1}PC(\theta)\mathbf{1}, \quad (1.15)$$

which uniquely characterizes the SDF and the physical probabilities in terms of the Arrow-Debreu prices and the wealth-to-consumption ratios:

$$\begin{aligned}\tilde{M} &= [\text{diag}(Q^{-1}PC(\theta)\mathbf{1})]^{-1}, \\ P &= PC(\theta)^{-1}Q \text{diag}(Q^{-1}PC(\theta)\mathbf{1}).\end{aligned}\tag{1.16}$$

Hence, given measurements of the price-dividend ratio on the wealth portfolio and the preference parameter θ , we can still recover the SDF and physical probabilities separately from Arrow-Debreu prices alone, without using macroeconomic data.

An important parameter which affects the the implied physical probabilities and the SDF is the preference parameter θ . When $\theta = 1$, the preferences collapse a to power utility, in which case the expressions for the implied physical probabilities and the stochastic discount factor reduce to their expected utility counterparts in (1.7)-(1.8). For $\theta \neq 1$, the identification of the probabilities and risk adjustments now depends on the magnitudes of the price-dividend ratios in each state. If the price-dividend ratios vary across the states, which we show is empirically relevant, this impacts the inference on the implied physical probabilities and the risk adjustments.

One of the qualitative implications of the recursive structure has to do with the impact of risk-free rates on the physical probabilities. Ross (2013) demonstrates that if risk-free rates are constant across the states, the physical probabilities reduce to the risk-neutral probabilities in the world with expected utility. This, however, is no longer the case with recursive preferences. Indeed, even when risk-free rates are constant, the variation in the price-dividend ratios is still going to impact the measurements of physical probabilities and create a wedge between them and the risk-neutral ones. With recursive preferences, the physical probabilities equal to the risk-neutral probabilities only when both the risk-free rates and wealth-to-consumption ratios do not vary across the states.

In the next section, we use a calibrated model to illustrate the quantitative importance of recursive preferences to identify the physical probabilities and risk adjustments. Then, we implement this framework in the data and argue that the market data supports a specification with preference for early resolution of uncertainty ($\psi > 1, \theta < 1$).

2 Economic Model

2.1 Economy Dynamics

Consider a discrete-time, infinite horizon endowment economy. The representative agent has recursive utility over future consumption described in (1.9), which allows for a preference for the timing of resolution of uncertainty. The endowment growth λ_j follows a time-homogeneous Markov process with a transition matrix P , as in Weil (1989) and Mehra and Prescott (1985). Notably, in our model economic growth rates and asset returns are stationary, while the levels of consumption and asset prices follow a random walk process.

Given our specification of the endowment dynamics and the agent's preferences, we can characterize the equilibrium stochastic discount factor m_{ij} in (1.10), and use a standard Euler equation,

$$E_i m_{ij} R_{ij} = 1, \quad (2.1)$$

to compute the equilibrium prices of financial assets, such as Arrow-Debreu assets, the wealth portfolio and the risk-free bonds. To evaluate model implications for the equity market, following Abel (1990) we model equity as a leveraged claim on aggregate consumption and specify its dividend dynamics in the following way:

$$\lambda_j^d = 1 + \mu_d + \phi(\lambda_j - \mu - 1), \quad (2.2)$$

where μ_d is the mean dividend growth and $\phi > 1$ is the dividend leverage parameter. For simplicity, we abstract from separate equity-specific shocks, so that aggregate dividend growth is perfectly correlated with consumption growth.

2.2 Model Calibration

The model is calibrated on a quarterly frequency to match the key features of U.S. real consumption growth and financial asset market data from 1929 to 2010. Specifically, we first start with AR(1) dynamics of consumption growth on a quarterly frequency:

$$\lambda_{t+1} = 1 + \mu + \rho(\lambda_t - \mu - 1) + \sigma\epsilon_{t+1}, \quad (2.3)$$

where ϵ_{t+1} is a Normal shock. We calibrate the mean μ , persistence ρ and volatility σ parameters so that a long sample of consumption growth simulated from the above specification

and time-aggregated to an annual frequency targets the mean, volatility and persistence of annual consumption growth in the data. To calibrate the dividend process, we set the average dividend growth to be the same as the mean consumption growth and fix the dividend leverage parameter to $\phi = 3$, similar to other studies. We then discretize the AR(1) dynamics of consumption growth into 3 states using the Tauchen and Hussey (1991) quadrature approach.³

The calibrated parameter values are presented in Table 1, and the key moments for consumption growth in the model and in the data are shown in Table 2. In the data, the consumption corresponds to annual real expenditures on non-durables and services from the BEA tables; in the model, the population moments are computed from a long simulation of a Markov process for quarterly consumption growth, time-aggregated to an annual horizon. As shown in Table 2, our calibration ensures that the model matches very closely the key properties of consumption growth in the data. Both in the model and in the data, the average consumption growth is 1.90%, and its persistence is about 0.50 on annual frequency. The volatility of consumption growth is 2.50% in the model, close to 2.20% in the data.

We calibrate the preference parameters δ, γ and ψ to match the key moments of the financial asset market data. The subjective discount factor δ is set at 0.988, annualized. The risk aversion is calibrated to $\gamma = 25$ and the inter-temporal elasticity of substitution parameter ψ is equal to 2.⁴ As shown in Table 2, our model delivers an average market price-dividend ratio of about 60, a market risk premium of 7% and the risk-free rate of 1.5%, which is consistent with the data. The implied volatility of the market return is about 10% in the model, which is lower than its typical estimates in the data of 15 – 20%. Recall that for simplicity, our model does not entertain equity-specific shocks in the dividend dynamics, which can help match the volatility of dividends and returns in the data.

Notably, as in the long-run risks literature, we focus on the case when both the inter-temporal elasticity of substitution and risk aversion parameters are above one, so that $\theta = (1 - \gamma)/(1 - 1/\psi) = -48$ is below zero. As discussed in Bansal and Yaron (2004), this parameter configuration is economically appealing for several reasons. First, keeping $\psi > 1$ ensures that the substitution effect dominates wealth effect, which implies that the equilibrium equity valuations fall in bad times of low economic growth. For example, in our calibration, the market price-dividend ratio is 62.77 in the best consumption growth state (state 1), and it drops to 57.12 in the worst state (state 3), and similar for the wealth

³Our main results are virtually unchanged using alternative numbers of economic states, or specifying the model on an annual frequency.

⁴Our risk aversion coefficient is higher than in Bansal and Yaron (2004) who use a monthly calibration of the model. Note that our model is specified on a quarterly frequency and does not entertain a separate expected growth component. See Bansal et al. (2011) for further discussion of time-aggregation issues in measurements of the preference parameters.

portfolio. Further, under such a parameter configuration the agent has preference for early resolution of uncertainty ($\psi > 1/\gamma$), and thus dislikes negative shocks to expected consumption or increases in aggregate volatility. These model predictions are directly supported in the data, and motivate a preference parameter configuration where $\theta < 0$.⁵

2.3 Model Implications: Consumption States

To highlight the main intuition for our results, we first consider the identification of the physical probabilities and the risk adjustments in the benchmark model specification using the consumption states. In the next section, we present the analysis under the market return-based states.

We use our model to compute the equilibrium Arrow-Debreu prices Q between the three consumption states, and further decompose them into the physical probability P and the risk adjustment through the stochastic discount factor M :

$$\underbrace{\begin{bmatrix} 0.47 & 0.50 & 0.03 \\ 0.07 & 0.61 & 0.33 \\ 0.01 & 0.17 & 0.82 \end{bmatrix}}_Q = \underbrace{\begin{bmatrix} 0.68 & 0.31 & 0.01 \\ 0.17 & 0.67 & 0.17 \\ 0.01 & 0.31 & 0.68 \end{bmatrix}}_P \times \underbrace{\begin{bmatrix} 0.69 & 1.60 & 3.44 \\ 0.39 & 0.91 & 1.96 \\ 0.24 & 0.56 & 1.21 \end{bmatrix}}_M, \quad (2.4)$$

where \times indicates element-by-element multiplication. Note that the SDF depends both on the current and future states. The SDF value is highest going from the best state 1 which has the highest consumption growth to the worst state 3 in which consumption growth is lowest, and smallest values of SDF obtain when we transition from the worst 3 to the best consumption state 1. The Arrow-Debreu prices incorporate the effects of both the risk adjustment and the physical transition probabilities. In this case, because going from the best to the worst state is very unlikely, the Arrow-Debreu price between states 1 and 3 is relatively inexpensive. On the other hand, consumption in the worst state next period is very valuable given current worst state, which can be attributed both to a relatively high risk compensation for remaining in state 3 and the persistence of the Markov chain.

The Arrow-Debreu price decomposition above is based on the equilibrium solution of the model given the full calibration of the endowment dynamics and preference parameters. Let us now consider the case when the researcher only has access to the model-generated data on Arrow-Debreu prices and the wealth-to-consumption ratios and tries to identify the physical probabilities of economic states and the implied risk compensation. Following our

⁵See Bansal et al. (2005) and Hansen et al. (2008) for the cross-sectional evidence, Bansal et al. (2007) and Bansal et al. (2011) for the empirical evaluation of the model.

discussion in Section 1.3, the identified values are based on the preference parameter θ , so we consider a range of possible values for θ and use the conditions in (1.16) to recover the implied physical probabilities and SDF at the entertained values of θ .

In the top two panels of Figure 1 we show the implied unconditional probability of being in the bad state, and the implied value for the SDF which corresponds to transitioning from the good to the bad state relative to remaining in the good state (i.e. $\frac{m_{1,3}}{m_{1,1}}$). When θ is equal to its calibrated value of -48 , the unconditional probability of remaining in the bad state and the relative value of the SDF in the bad state are equal to their equilibrium values of 25% and 5.02, respectively. When the candidate value of θ is smaller in the absolute value relative to its calibrated value, the recovery is based on the understated magnitude of the risk compensation, leading to biased estimates of physical probabilities and risk adjustments. Specifically, the Figure shows that the probabilities of bad events are biased upwards, while the risk adjustments of bad events are biased downwards.

Consider, for example, the decomposition of the Arrow-Debreu prices into the implied physical probabilities and implied SDF under the assumption of expected utility, so that θ is set to 1:

$$\underbrace{\begin{bmatrix} 0.47 & 0.50 & 0.03 \\ 0.07 & 0.61 & 0.33 \\ 0.01 & 0.17 & 0.82 \end{bmatrix}}_Q = \underbrace{\begin{bmatrix} 0.47 & 0.50 & 0.03 \\ 0.07 & 0.61 & 0.33 \\ 0.01 & 0.17 & 0.82 \end{bmatrix}}_{P(\theta=1)} \times \underbrace{\begin{bmatrix} 0.99 & 1.00 & 1.00 \\ 0.99 & 1.00 & 1.00 \\ 0.99 & 1.00 & 1.00 \end{bmatrix}}_{M(\theta=1)}. \quad (2.5)$$

Under expected utility, the SDF does not depend on the current state, so all the rows in the matrix $M(\theta = 1)$ are identical. Further, as can be seen from the above equation, under expected utility there is barely any variation in the SDF across future states, compared to the benchmark SDF in (2.4). This is consistent with the broad literature which finds that time- and state-independent expected utility models can not generate enough volatility of the SDF to account for the asset market prices; for example, the risk-free rate and the equity premium puzzles imply that standard expected utility models cannot simultaneously explain the levels of the risk-free rate and equity risk premium given the actual dynamics of consumption growth in the data (see e.g., Mehra and Prescott (1985)). Following the fact that under the expected utility assumption there is very little action coming from the risk compensation, virtually all the difference in the Arrow-Debreu prices is now attributed to the difference in the implied physical probabilities. To account for the observed asset market features, the recovery framework needs to twist the physical dynamics of the endowment, and in particular, it needs to put more weight on the likelihood of bad events to generate expensive Arrow-Debreu prices of going into bad states. Because of that, under expected

utility the unconditional probability of bad states is significantly biased upwards and equal to 62% versus its true value of 25%, as shown in Figure 1. Similar biases for the stochastic discount factor and physical probabilities arise at alternative values of θ which are less than the calibrated θ in absolute value. These values of the preference parameter underestimate the volatility of the SDF and the amount of risk compensation it can generate, and thus lead to higher implied probabilities of bad events, as shown in Figure 1.

The mis-specification of the economic dynamics has important implications for the implied moments of both macroeconomic and financial market variables, as shown in the last column of Table 2 where we document the moments of consumption growth and stock returns computed using the recovered physical probabilities under the expected utility assumption. As evident in the Table, attributing more likelihood to bad events leads to a significant downward bias of the measured average consumption growth rate and mean returns. For example, while the mean consumption growth rate is calibrated to 1.9%, using the recovered physical probabilities under the expected utility framework, the implied mean is -0.8%. Similarly, as shown in the bottom panel of Figure 1, the average return on the market implied by the physical probabilities under expected utility is less than 1%, relative to its calibrated value of 8.9%.

To formally evaluate the mis-specification of the economic dynamics, we compute the Kullback-Leibler (KL) divergence between the implied and calibrated physical probabilities. This is the standard measure of the fit of distributions, and is calculated as the distance between the true distribution and a candidate distribution; smaller values of KL divergence imply a better fit. It is given by the following equation:

$$KL(P||P_\theta) = \sum_i \pi_i \left[\sum_j p_{i,j} \log \left(\frac{p_{i,j}}{p_{i,j}^\theta} \right) \right], \quad (2.6)$$

where π_i are the unconditional probabilities of being in each state, $p_{i,j}$ are the calibrated physical transition probabilities, and $p_{i,j}^\theta$ are the recovered transition probabilities for a particular candidate value of θ . We plot the value of KL divergence for candidate values of θ on the bottom right panel of Figure 1. The criterion function is minimized to zero (no deviation from the true distribution) at the true value of θ , and significantly rises for alternative values of θ .

Overall, our findings suggest that while analysis based on the expected utility framework suggests a high probability of bad events (e.g., disasters), these results have to be interpreted with caution and might just indicate a mis-specification of the underlying preference structure of the agent. Indeed, when the preference structure allows for a preference for the

timing of the resolution of uncertainty, the burden of explaining the cross-section of Arrow-Debreu prices falls less on the physical probabilities, and the differences in Arrow-Debreu prices are attributed more to variations in risk compensation across states.

2.4 Model Implications: Market Return States

In the previous Section, we considered consumption growth regimes to identify the aggregate economic states and decompose the Arrow-Debreu prices for these states into physical probabilities and risk adjustments. However, using the observed index option prices in the data, we can only identify the risk-neutral distributions and the Arrow-Debreu prices corresponding to the states of aggregate market returns. In this section, we use our model simulation to analyze the implications of the alternative identification of the states based on market returns for the recovery of preference parameters and physical probabilities.

Specifically, given the assumption of the three underlying consumption states, the distribution of realized market returns consists of 9 possible return values, R_{ij} , for $i, j = 1, 2, 3$. We identify the best return state corresponding to the top third of the distribution of market returns; in our calibration, the best return state includes all three returns going to the best consumption state, and the return from the worst to the middle consumption state. The worst return state corresponds to the bottom third of the distribution of returns, and includes all three returns going to the worst consumption state. Finally, the middle return state includes the intermediate values of returns. We compute the physical probability and the risk-neutral probability matrices, average values of wealth-to-consumption ratio and the risk-free rate, and the Arrow-Debreu prices for these return-based states. We then use the recovery methodology to decompose the Arrow-Debreu prices into the implied physical probabilities and risk adjustments, as in the previous section.

Overall, as the return-based states generally co-move with the consumption-based states, return-based state recovery leads to similar conclusions for the importance of the recursive utility to correctly identify the physical probabilities and the risk adjustments. However, the added noise and averaging out in the measurement of the aggregate states through returns tends to decrease the persistence of the states and diminishes the variability of the Arrow-Debreu prices across states, which brings down the magnitude of the implied risk compensation relative to the calibrated model.

Indeed, the transition matrix for the return states is given by,

$$P^r = \begin{bmatrix} 0.56 & 0.16 & 0.28 \\ 0.17 & 0.67 & 0.17 \\ 0.28 & 0.16 & 0.56 \end{bmatrix}, \quad (2.7)$$

and features a much lower persistence of 0.27 than 0.50 for the consumption states. The Arrow-Debreu prices for the return states also have less variation across the states, relative to the consumption states, and are given by:

$$Q^r = \begin{bmatrix} 0.18 & 0.43 & 0.38 \\ 0.07 & 0.61 & 0.33 \\ 0.17 & 0.02 & 0.81 \end{bmatrix}. \quad (2.8)$$

Next we decompose the Arrow-Debreu prices for the return states into the implied physical probabilities and the risk adjustments for alternative values of θ , following the approach outlined in the previous section, and report the results in Figure 2. Similar to our previous findings, positive values of θ significantly overestimate the probability of bad states, underestimate the risk adjustments for bad states, and lead to substantial negative biases for the average returns. The preference structure which brings these magnitudes close to the calibrated values relies on strongly negative values of θ . Notably, as the return identification is not based on the consumption states, the parameter which minimizes the KL distance between the calibrated and implied physical probabilities no longer results in a perfect match of the two probabilities, and is different from the calibrated value. Indeed, due to less variation in Arrow-Debreu prices for the return based states, the implied magnitude of risk compensation is lower, and the value of θ of -12 is smaller than for the consumption-based states. Thus, while the return-based identification leads to similar conclusions for the importance of $\theta < 0$ to account for the asset prices, the added noise and averaging out in the measurement of aggregate states by using return-based states biases down the overall magnitude of the preference parameter θ .

3 Empirical Analysis

3.1 Data

We use the OptionMetrics database to obtain daily closing prices for exchange-traded S&P 500 index options on the CBOE from 1996 to 2011. On each trading day, there are an average of 840 put and call options contracts written on the S&P 500 index and differing

with respect to the expiration date and the strike price; however, a significant number of the contracts are subject to liquidity concerns, such as zero trading volume and large bid-ask spreads. To mitigate possible microstructure issues, we follow Figlewski (2008) to apply standard data filters and exclude contracts with zero trading volume, quotes with best bid below \$0.50, and very deep-in-the-money options. Further, our benchmark analysis is conducted on a quarterly frequency, where we use options with 3 months to maturity and track their prices on the expiration dates of the contract. We focus on the quarterly frequency for several reasons. First, the main liquidity in the options markets lies in the primary quarterly expiration cycle: the main hedging instruments for the options are the S&P 500 futures which feature quarterly expirations, so the majority of S&P 500 options also trade on the primary quarterly cycle with expirations in March, June, September, and December of each year. Second, in our empirical analysis, aggregate economic states are identified from the distribution of market returns, and focusing on a relatively lower quarterly frequency helps to reduce non-systematic noise in prices. Finally, using quarterly frequency in the data allows us to directly relate our findings to the economic model in Section 2. We have verified that our findings are robust to using a monthly data horizon, and we report the results in Section 4. In addition to options prices, we use data on interest rates which correspond to the 3 month U.S. Treasury rate, and the returns and price-dividend ratio on the S&P 500 index.

As the options data and the implied risk-neutral probabilities are based on the S&P 500 index, we use the distribution of the capital gains on the index, $r_{t+1}^m = \log \frac{S_{t+1}}{S_t}$, to identify the aggregate state of the economy. As evident from the histogram of the capital gains in Figure 3, the return distribution is fat-tailed and negatively skewed. Indeed, the skewness of capital gains over the 1996-2011 period is -0.73, and its kurtosis is 4.62 on a quarterly frequency, which is higher than for a normal distribution. Large negative moves in quarterly returns are likely to contain important information about the aggregate economy and are in general more important from the perspective of a risk-averse investor relative to large positive shocks in returns. Motivated by such considerations, we identify 3 economic states, good, medium, and bad, where the good state corresponds to the upper 50% percentile of the return distribution, the bad state represents the lowest 25% of the returns, and the medium state is in between. The median return in each bin identifies the level of return in each of the economic states, and is given by:

$$\begin{bmatrix} r_1^m & r_2^m & r_3^m \end{bmatrix} = \begin{bmatrix} 22.63\% & 0.19\% & -35.51\% \end{bmatrix}, \quad (3.1)$$

annualized. The estimated transition matrix for the states in the data is equal to,

$$P = \begin{bmatrix} 0.53 & 0.34 & 0.13 \\ 0.47 & 0.20 & 0.33 \\ 0.44 & 0.13 & 0.44 \end{bmatrix}. \quad (3.2)$$

Because the good state corresponds to the upper 50% of the return distribution, there is a considerable probability of remaining in the good state (53%) or transitioning to the medium one (34%). The overall persistence of the aggregate state implied by the transition matrix is low and matches the persistence of returns in the data. The persistence of returns in the data is 0.13, while the persistence of the estimated Markov chain above is 0.23. The persistence of the return states is also very similar to the value in the economic model of 0.27, as discussed in Section 2.4.

We verify that the states identified by the returns on the index contain meaningful information about the aggregate economy, and we report the average values of the key economic variables, such as real consumption growth, VIX and asset prices, in Table 3. As shown in the Table, there is a significant difference between the average economic variables in the two extreme states. The median real consumption growth is 1.79%, annualized, in the good state, while it is a much lower 0.29% in the bad state. The VIX index, which measures uncertainty about the market, is 20.22 in the good state, relative to a much higher value of 30.29 in the bad state, and the price-dividend ratio for the index increases from about 50 to 55 going from the bad state to the good state. Looking at Figure 4, we see that the implied volatility curves for each state generated from options data are increasing across the range of moneyness as the aggregate state worsens. In particular, at-the-money implied volatility increases from 18% in the good state to 26% in the bad state. Overall, our economic states meaningfully capture the real growth and uncertainty prospects in the aggregate economy.

3.2 Estimation of the Risk-Neutral Distribution

Theoretically, the entire risk-neutral probability distribution can be extracted directly using a continuum of options contracts, as shown in Breeden and Litzenberger (1978). Let $\tilde{P}(x) = \int_{-\infty}^x \tilde{p}(z) dz$ denote the risk neutral cumulative distribution function, K be the strike price,

and r the risk-free interest rate. Then, given the current value S of the underlying, the price of a European call option expiring at time T is given by:

$$\begin{aligned} C(K; S, r) &= PV\{\mathbb{E}^Q[\max(S_T - K, 0)]\} \\ &= \int_K^\infty e^{-rT} (S_T - K) \tilde{p}(S_T) dS_T. \end{aligned}$$

Differentiating the call price with respect to the strike price allows us to relate the risk-neutral distribution to the prices of call options:

$$\frac{\partial C}{\partial K} = e^{-rT} \left[-(K - K) \tilde{p}(K) + \int_K^\infty -\tilde{p}(S_T) dS_T \right] = -e^{-rT} [1 - \tilde{P}(K)], \quad (3.3)$$

so that the risk-neutral probability is determined by the second derivative of the price of call options:

$$\tilde{p}(K) = e^{rT} \frac{\partial^2 C}{\partial K^2}. \quad (3.4)$$

In practice, we do not observe the entire continuum of options prices, and we do not observe very deep in- and out-of-the money contracts to capture the tails of the distribution. To address the first issue, we interpolate the data to fill in the quotes between listed strikes. Following Shimko (1993), we first transform option prices into Black and Scholes (1973) implied volatilities and interpolate the implied volatility surface, and then transform the interpolated curve back to find a theoretical profile of call option prices by strike. As we are interested in the conditional probabilities of being in the good, medium and bad aggregate states, we use quarterly data to calculate the average volatility surface in each of the three states and then compute the implied risk-neutral distributions conditional on each state. To deal with the unavailability of deep in- and out-of-the money contracts, we follow the steps in Figlewski (2008) and approximate the tails of the risk-neutral density by a Generalized Extreme Value (GEV) distribution. The tail parameters of the distribution are chosen to match the curvature of the risk-neutral probability density function at two extreme points, along with the requirement that the tail probabilities in both the observed risk-neutral distribution and the GEV tail distributions must equal. A similar approach is also pursued by Vilkov and Xiao (2012). All of the details for the estimation of the risk-neutral distribution are provided in the Appendix.

Figure 5 shows the estimated risk-neutral distributions for each of the aggregate states, together with the GEV adjustments of the right and left tails of the distribution. The bottom panel of Table 3 summarizes the conditional moments of the risk-neutral distribution. Going from good to bad state, risk-neutral volatility increases from 18.92% to 25.27% ; the risk-neutral 3rd central moment becomes about two times more negative, and the 4th

moment of the distribution increases twofold as well. Overall, good states are characterized by relatively lower volatility and a relatively smaller left tail, which is consistent with our findings on the behavior of VIX and asset prices in the previous section. Our evidence is also consistent with Chang, Christoffersen, and Jacobs (2012) who find that risk-neutral skewness correlates negatively with market returns.

3.3 Data Implications for Probabilities and Risk Adjustments

Using our estimates of the risk-neutral distributions, we can compute the conditional risk-neutral probabilities between the three aggregate states implied by the options prices. The risk-neutral probabilities, adjusted by the risk-free rates, allow us to calculate the matrix of Arrow-Debreu prices which is specified below:

$$Q = \begin{bmatrix} 0.47 & 0.14 & 0.38 \\ 0.48 & 0.13 & 0.38 \\ 0.50 & 0.09 & 0.41 \end{bmatrix}. \quad (3.5)$$

The Arrow-Debreu prices appear relatively low for bad states: for example, the Arrow-Debreu price of going from the good state to the bad state is 0.38, relative to 0.50 for going from the bad state to the good state. Ex-ante, it is not clear whether the difference in these prices is attributable to the difference in physical transition probabilities (as suggested by our estimate in (3.2), since going from good to bad is less likely than going from bad to good), or by the difference in the magnitudes of risk compensation between the states. To separate the Arrow-Debreu prices into the implied physical probabilities and the risk-adjustment, we implement our market-based recovery methodology outlined in Section 1, allowing for recursive state-dependent utility and a preference for the timing of the resolution of uncertainty.

The recovery of the probabilities and the SDF relies on measurements of the wealth-to-consumption ratio and the preference parameter θ . The wealth-to-consumption ratio is not directly observed in the data. Consistent with our economic model, we assume that the log wealth-to-consumption ratio is proportional to the log price-dividend ratio, $\log PC \approx \alpha + \beta \log PD$, and set the scale parameter β to match the volatility of the wealth-to-consumption ratio in the model relative to the volatility of the price-dividend ratio over the long historical sample in U.S.. Given our economic model, the implied estimate of $\beta \approx 1\%$, which is consistent with empirical findings in Lustig, Nieuwerburgh, and Verdelhan (2012) that the wealth-to-consumption ratio is less volatile than the price-dividend ratio. We examine the robustness of our results to the scale parameter β in Section 4.

Given these measurements of the wealth-to-consumption ratio, we entertain a range of possible preference parameters θ and identify the implied physical probability distribution and the SDF for each of the values of this parameter. For all the values of θ , the top panels of Figure 6 depict the implied unconditional probability of being in the bad state and the implied value of the SDF for transitions from the good to the bad state, while the bottom panels show the implied average market returns, computed under the implied physical probabilities, and the Kullback-Leibler divergence criterion between the recovered transition probabilities and estimated transition probabilities in the data. The actual estimates for the probability of being in bad states, the value of the SDF and the implied average market returns in the case of early resolution of uncertainty and the expected utility are provided in Table 4.

Our empirical findings based on the options data are consistent with the output of the economic model in Section 2. As shown in the bottom right panel of Figure 6, the implied and actual physical probabilities of the states are best matched when θ is sufficiently negative, and the the Kullback-Leibler divergence criterion is minimized at $\theta = -11.28$, which is very close to the model value using the return-based states.

Measurements of θ have important implications for the recovery of physical probabilities and risk adjustments. As in the economic model, the recovered probability of the bad state is significantly higher when the preference parameter θ is positive or not sufficiently negative. Indeed, for $\theta = 1$ the preferences collapse to expected utility and the implied probability of the bad state is about 60%, compared to its set value of 25%. When θ equals its Kullback-Leibler optimal value of -11.28, the representative agent has a strong preference for early resolution of uncertainty, and the recovered physical probability of the bad state is close to the actual value in the data. The economic channel which accounts for the upward bias in the recovered probability of bad events is the one highlighted in the economic model: expected utility features very little risk adjustment across states, so physical probabilities have to bear all the burden of explaining the cross-section of Arrow-Debreu prices in the data.

To further illustrate the importance of the recursive utility structure, consider the decomposition of the Arrow-Debreu prices into implied physical probabilities and the SDF,

under the expected utility framework and under a framework which features a preference for early resolution of uncertainty,

$$\begin{aligned}
 \underbrace{\begin{bmatrix} 0.47 & 0.14 & 0.38 \\ 0.48 & 0.13 & 0.38 \\ 0.50 & 0.09 & 0.41 \end{bmatrix}}_Q &= \underbrace{\begin{bmatrix} 0.55 & 0.18 & 0.28 \\ 0.55 & 0.17 & 0.28 \\ 0.58 & 0.12 & 0.30 \end{bmatrix}}_{P(\theta=-11.28)} \times \underbrace{\begin{bmatrix} 0.87 & 0.78 & 1.36 \\ 0.87 & 0.79 & 1.37 \\ 0.86 & 0.77 & 1.35 \end{bmatrix}}_{SDF(\theta=-11.28)} \\
 &= \underbrace{\begin{bmatrix} 0.26 & 0.17 & 0.57 \\ 0.27 & 0.16 & 0.57 \\ 0.28 & 0.11 & 0.61 \end{bmatrix}}_{P(\theta=1)} \times \underbrace{\begin{bmatrix} 1.79 & 0.82 & 0.67 \\ 1.79 & 0.82 & 0.67 \\ 1.79 & 0.82 & 0.67 \end{bmatrix}}_{SDF(\theta=1)}.
 \end{aligned} \tag{3.6}$$

where, again, \times denotes element-by-element multiplication.

In the benchmark case featuring early resolution of uncertainty, the implied physical probabilities are close to their estimates in the data, and the SDF correctly identifies bad states as the ones with the highest risk compensation. This is consistent with the intuition from our economic model, as shown in (2.4). Notably, there is less variation in the SDF across states, which can be explained by a lower persistence of the aggregate states in the data relative to the economic model. In the case of expected utility, implied physical probabilities are quite different from their estimates in the data. The recovered probability of bad events are so large that the implied risk compensation in bad states is actually considerably smaller than in good states: the magnitude of the SDF going to the bad state is more than two times smaller than going to the good state, while the opposite is the case for recursive utility. To obtain the economically plausible implication that the bad state requires higher risk compensation in the data than the good state, the utility structure should incorporate a sufficient degree of preference for early resolution of uncertainty. In our case, this requires θ to be below -8.

The measurements of the physical probabilities have direct implications for the moments of stock returns and macroeconomic variables. As we show in the bottom left panel of Figure 6, using the implied physical probabilities under expected utility leads to negative estimates of average returns of about -15%. This is a direct consequence of assigning a large probability to bad states with low negative returns. Using negative values of $\theta = -11.28$ results in a more plausible estimate of average returns of about 3%, which is more consistent with the evidence in the data. We further evaluate the implications for the measurements of physical probabilities on higher order moments of returns. The measurements of physical probabilities do not have a significant effect on the implied physical volatility of returns: it is 12.92% under expected utility, relative to 12.61% under early resolution of uncertainty

with θ of -11.28. However, both skewness and kurtosis of returns are closer to the actual data under the recursive utility structure: for example, the return skewness is about -0.7 for $\theta = -11.28$ relative to 0.6 under the expected utility.

In sum, consistent with our economic model, our empirical findings suggest that the recovery of physical probabilities and the SDF is significantly affected by the underlying preference for the timing of the resolution of uncertainty. When the magnitude of the preference for early resolution of uncertainty is not fully accounted for, the implied physical probabilities tend to trade-off higher probabilities of bad economic events for lower risk compensation in these events. A specification with enough preference for early resolution of uncertainty matches the actual physical probabilities and the moments of returns in the data quite well.

4 Robustness

4.1 Measurements of Wealth-to-Consumption Ratio

In our benchmark case, we estimate the log wealth-to-consumption ratio assuming it is proportional to the log price-dividend ratio, $\log PC \approx \alpha + \beta \log PD$, and the coefficient β is identified from the volatility of the wealth-to-consumption ratio in the model relative to the volatility of price-dividend ratio in the data. To check the robustness of our findings, we consider alternative values of β , and plot the implied physical probability of bad states for a range of β and θ values on Figure 7. As is shown in the plot, to match a calibrated probability of the bad state of 0.25 for values of $\beta > 0$, which is the economically plausible case, the implied preference parameter θ should be below one. While the actual value of θ depends on the choice of β , for all configurations of $\beta > 0$ the implied preference structure suggests a preference for early resolution of uncertainty.

4.2 Measurements of Aggregate States

Our benchmark specification features three economic states which are identified using the 25th and 50th percentile cut-offs for the return distribution in the data. We consider various robustness checks with respect to the location and the number of bins, and we report the results for the recovery of physical probabilities and the SDF under alternative identifications of the aggregate states in Table 4.

Specifically, we first entertain the case where the cut-off point for the bad state corresponds to the 20th percentile of the return distribution. As shown in Table 4, the recovery under expected utility still over-estimates the probability of the bad state to be 40% which results in a -8.47% estimate for the average market return. The value of $\theta = -11.28$ which corresponds to the preference for early resolution of uncertainty minimizes the KL divergence criterion and leads to positive average market returns and the bad state probability closer to the data. Similarly, when the left tail is set above our benchmark specification to the 30th percentile of the distribution of market returns, the implied probability of the bad state is 50% and average market returns are negative under the expected utility. On the other hand, the implied probability of the bad state is 32% while average market returns are 5.7% under the recursive utility structure with preference for early resolution of uncertainty. Note that in both of these configurations we can still meaningfully identify the aggregate economic states. For example, for the 20th percentile left tail specification, the PD ratio in the best state is 55 relative to 52 in the worst state, consumption growth in the best state is 1.8% versus 0.3% in the worst state, and the average VIX is 20 in the best state compared to 35 in the worst state. Similar results hold for the 30th percentile for the specification of the left tail. Interestingly, as the bad state corresponds more and more to the tail events (20th percentile compared to 25th and 30th percentile), the implied value of θ becomes more negative. This suggests that the recursive utility structure and the preference for early resolution of uncertainty play an increasingly important role to account for the larger tail events in stock markets; see e.g. Drechsler (2013), Eraker and Shaliastovich (2008) and Shaliastovich (2012) for the discussion of equilibrium recursive utility models with jumps.

While our benchmark specification is a three state model, similar results hold for a two state model as well, which we report in Table 4. We consider as a robustness check a two-state specification where the left tail (bad state) is defined as, respectively, the 20th, 25th, and 30th percentiles of quarterly returns. For all cases, an expected utility specification over-estimates the probability of bad states, and in all the cases except the 30% cut-off, the implied average market returns are negative. The value of θ that minimizes KL divergence between the recovered conditional distribution and the transition probabilities in the data are all negative and imply a preference for early resolution of uncertainty. Under recursive utility, both the probability of bad states and average market returns are much closer to the data compared to the case of expected utility.

We have also considered the robustness of our results with respect to alternative specifications for the good state. Generally, if the bin structure permits us to meaningfully identify aggregate economic states, our results remain robust.⁶

4.3 Monthly Horizon

While our main results are presented on a quarterly frequency, our results are robust to using a monthly horizon as well.

We use options with one month to maturity and track their prices on the same expiration cycle as in our benchmark specification, that is, we use quarterly observations in March, June, September, and December. We construct the risk-neutral distribution implied by the monthly options prices and compute Arrow-Debreu prices following our discussion in Section 1. Economic states are defined using the capital gains to the index over the past month and are binned at the 25th and 50th percentiles, as in our benchmark setup.

We report the results for the recovery of physical probabilities and the SDF for monthly horizons in Table 4, and show the implied unconditional probability of being in the bad state, the implied value of the SDF for transitions from the good to the bad state, the implied average market returns and the Kullback-Leibler divergence criterion for each value of θ in Figure 8. As seen in the Figure and the Table, the Kullback-Leibler divergence is minimized at $\theta = -39.46$. Under expected utility, the probability of the bad state is significantly biased upwards and is equal to 62%, and the implied average market return is very negative. Under a preference for early resolution of uncertainty, the probability of the bad state is 31% and the average market return is 6%. Overall, our evidence at the monthly horizon is similar to the benchmark specification and the economic model.

5 Conclusions

We show how to separately recover physical probabilities and risk adjustments from risk-neutral probabilities without using macroeconomic variables and allowing for a preference for the timing of the resolution of uncertainty, extending the recovery framework of Ross (2011) to the Kreps and Porteus (1978) recursive preferences of Epstein and Zin (1989) and Weil (1989). We implement our market-based recovery framework using S&P 500 options and find that the data strongly support a specification of early resolution of uncertainty, with

⁶For a sufficiently high cut-off point used to define a good state, the relative magnitude of economic variables in good states versus bad states are reversed compared to the benchmark case. This might be due to microstructure and data issues associated with the right tail of the return distribution.

preference parameter values similar to common values in the literature. Using the data and model simulations, we document significant biases in estimating physical probabilities and risk adjustments when the preference for early resolution of uncertainty is not sufficiently accounted for.

To highlight the implications of the timing of the resolution of uncertainty for the physical probabilities and risk adjustments, we first use a Mehra and Prescott (1985)-Weil (1989) economic model, which incorporates Epstein-Zin utility. We calibrate our model to match the stylized facts of financial market returns such as the equity premium and average risk-free rates. In the model, we see that failing to sufficiently account for a preference for early resolution of uncertainty leads to biased estimates of the physical distribution of returns, because we will attribute too large a proportion of the high state prices of bad states to physical likelihoods rather than risk-adjustment.

We then implement our market-based recovery approach using S&P 500 options data. We extract the risk-neutral distribution from options prices using a standard technique, and identify our economic states based on market returns. We apply the framework to the extracted risk-neutral distribution and recover implied dynamics for the physical return probabilities of the U.S. market. We show that not fully accounting for the preference for early resolution of uncertainty results in an over-estimation of the probabilities of bad states and downward-biased estimates of average returns. In all, the evidence from the S&P 500 index options market suggests that the representative agent for the U.S. economy has a strong preference for early resolution of uncertainty.

A Model Solution

The equilibrium price of the asset is computed using the Euler equation:

$$\mathbb{E}_i [M_{i,j} R_j] = 1. \quad (\text{A.1})$$

We first use this equation for the consumption asset. Given the expression for the stochastic discount factor in (1.12), we obtain that in equilibrium,

$$E_i \left[\delta^\theta \lambda_j^{1-\gamma} \left(\frac{PC_j + 1}{PC_i} \right)^\theta \right] = 1. \quad (\text{A.2})$$

This provides us the equation for the wealth-to-consumption ratio in each state i , and we solve the system of equations numerically using fixed point iteration.

Given equilibrium solutions to the wealth-to-consumption ratio, we can characterize the stochastic discount factor and obtain equilibrium prices of the Arrow-Debreu claims, the stock market, and the risk-free rates. Specifically, the Arrow-Debreu prices follow the equation (1.1). For the stock market claim, the Euler equation is given by,

$$E_i [M_{i,j} (PD_j + 1) (\mu_d + \phi(\lambda_j - \mu))] = PD_i, \quad (\text{A.3})$$

which leads to a linear matrix system for the price-dividend ratio of the market, PD_i . Risk-free rates satisfies

$$R_{f,i} = E_i [M_{i,j}]^{-1}, \quad (\text{A.4})$$

which can be solved using the equilibrium dynamics for the stochastic discount factor $M_{i,j}$.

B Estimation of Risk-Neutral Density

The exact steps for the estimation of the risk-neutral density from options prices are provided below:

1. Combine puts and calls, which are out-of-the-money (not too deep out-of-the-money, best bid at least \$0.50), and contracts not more than 20 points in-the-money.
2. Transform mid-prices into implied volatilities using Black and Scholes (1973). In the region of +/- 20 points from at-the-money, take a weighted average of put and call implied volatilities.
3. Fit a 4th order polynomial to the implied volatilities over a dense set of strike prices, and convert back into call option prices using Black-Scholes.
4. Numerically differentiate the call prices using (3.3) to recover the risk-neutral distribution function.

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Tables and Figures

Table 1: **Model Calibration**

Preferences	δ	γ	ψ
	0.988*	25	2
Consumption	μ	ρ	σ
	1.88*	0.28*	1.32*
Dividend	μ_d	ϕ	
	1.88*	3	

Calibration of model parameters. The model is calibrated on a quarterly frequency. The parameter values with superscript * are annualized, e.g. δ^4 and ρ^4 for the subjective discount factor and consumption growth persistence, 2σ for consumption volatility, and 4μ for the mean. The AR(1) dynamics of consumption growth is discretized into 3 states using the Tauchen and Hussey (1991) quadrature approach. Mean and volatility parameters are in percent.

Table 2: **Model Output**

	Data	Model	
		Under EZ	Under EU
$\mathbb{E}\left[\frac{C_{t+1}}{C_t}\right]$	1.90	1.90	-0.77
$\sigma\left[\frac{C_{t+1}}{C_t}\right]$	2.20	2.50	1.90
$\rho\left[\frac{C_{t+1}}{C_t}\right]$	0.50	0.50	0.44
$\mathbb{E}[PD_m]$	60.02	59.60	58.16
$\mathbb{E}[R_m]$	7.13	8.89	0.90
$\mathbb{E}[R_f]$	1.19	1.54	0.87

Data and model-implied mean, volatility, and persistence of annual consumption growth (top panel), and average price-dividend ratio, excess returns on the market and the risk-free rate (bottom panel). Data is annual from 1929 to 2010; model statistics are based on a long simulation of quarterly data time-aggregated to an annual horizon. “Under EZ” model output is based on the recursive utility configuration with preference for early resolution of uncertainty, while “Under EU” model output is based on the implied physical probabilities recovered under the expected utility assumption.

Table 3: **Economic Variables in Aggregate States**

	Mean	Std. Dev.	Econ. State		
			Good	Medium	Bad
Mkt Capital Gains	5.92	17.82	22.63	0.19	-35.51
Mkt PD ratio	57.81	14.00	54.50	57.29	49.54
VIX	22.31	8.08	20.22	18.77	30.29
Real cons. growth	1.31	0.83	1.79	1.74	0.29
RN Distribution:					
Volatility			18.92	19.57	25.27
3rd Moment×1000			-0.74	-0.84	-1.24
4rd Moment×1000			0.37	0.39	0.73

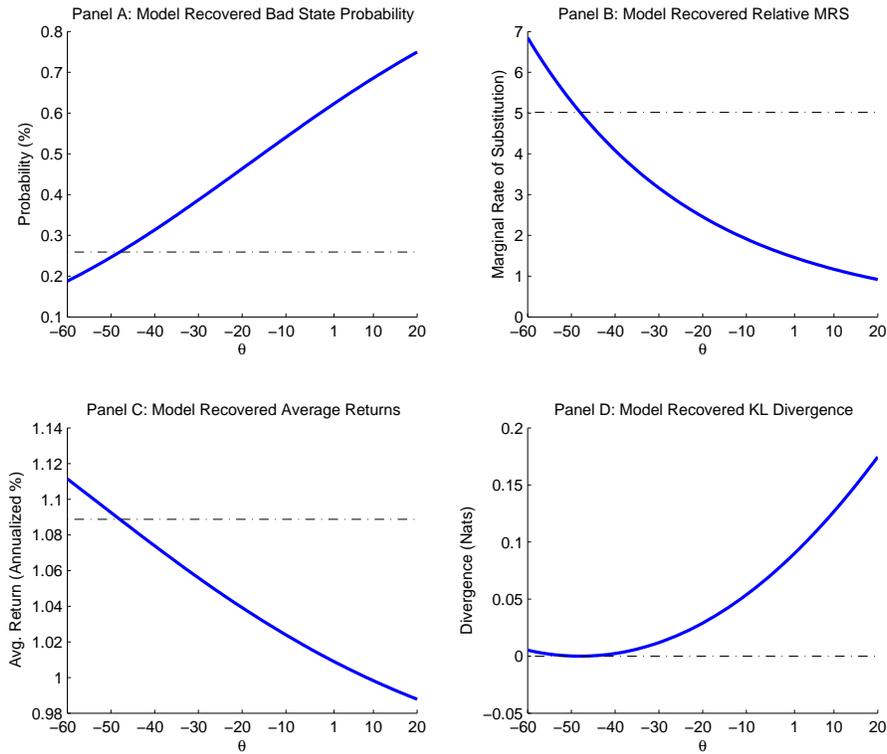
The top panel shows the mean and standard deviation of asset-price and macroeconomic variables, and their median values in good, medium, and bad economic states. Bottom panel shows the volatility, and 3rd and 4th centered moments of the risk-neutral distribution in each state. Economic states correspond to upper 50%, 25%-50%, and lower 25% distribution of capital gains of S&P 500, respectively. Quarterly observations from 1996 to 2011.

Table 4: **Implications for Probabilities and Risk Compensations**

	Under EZ				Under EU			
	θ	Pr. Bad	SDF	Mkt Ret	θ	Pr. Bad	SDF	Mkt Ret
Benchmark	-11.28	0.28	1.57	2.56	1.00	0.59	0.37	-14.85
Three States:								
<i>Left Tail:</i>								
20th pctile	-12.21	0.25	1.23	1.33	1.00	0.39	0.54	-8.47
30th pctile	-7.27	0.32	1.32	5.90	1.00	0.48	0.63	-0.62
Two States:								
<i>Left Tail:</i>								
20th pctile	-52.34	0.21	1.52	1.52	1.00	0.27	1.07	-2.15
25th pctile	-9.43	0.25	1.87	1.94	1.00	0.35	1.19	-2.88
30th pctile	-3.09	0.30	1.47	5.65	1.00	0.34	1.25	4.35
Monthly horizon	-39.46	0.31	1.12	6.07	1.00	0.62	0.22	-26.73

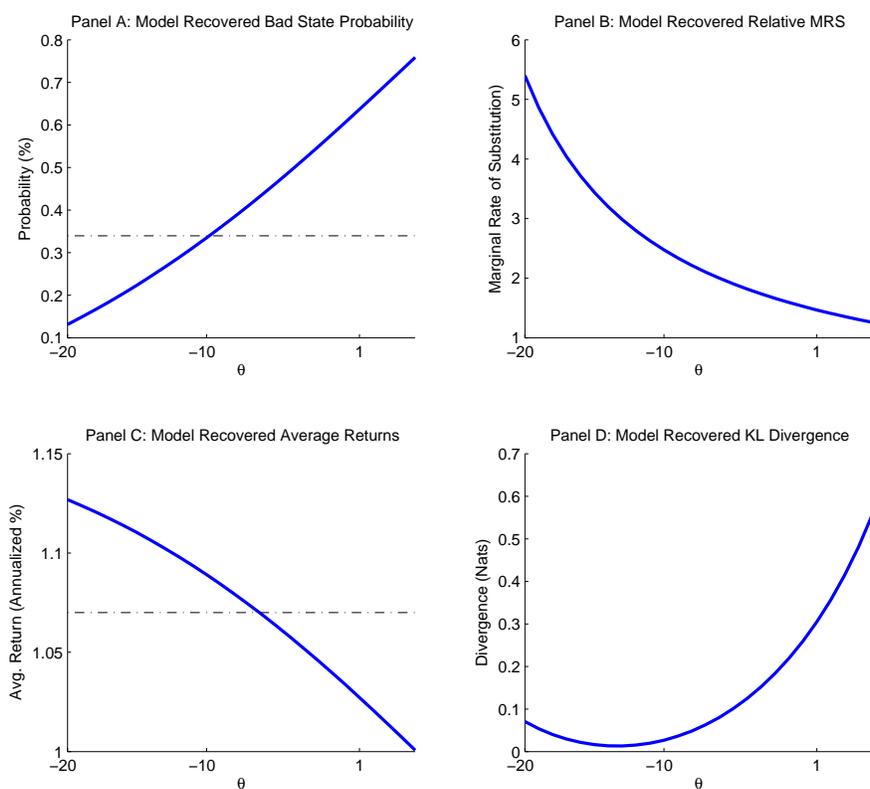
Implied physical probability of a bad aggregate state, the value of the stochastic discount factor from good to bad state relative to staying in a good state, and the implied average annualized market return, recovered under the specifications with recursive preferences (“Under EZ”) and under power utility (“Under EU”). Recursive preference model output corresponds to the optimal preference parameter θ which minimizes Kullback-Leibler divergence between the implied and physical conditional probabilities of aggregate states in the data; under expected utility, θ is fixed at 1. The benchmark setup features 3 states and the bad state cut-off at 25% of the return distribution. Robustness specifications include setting the bad state cut-off to 20th and 30th percentiles; using two state, and using monthly data horizons. Quarterly observations from 1996 to 2011.

Figure 1: Probabilities and Risk Adjustments: Economic Model



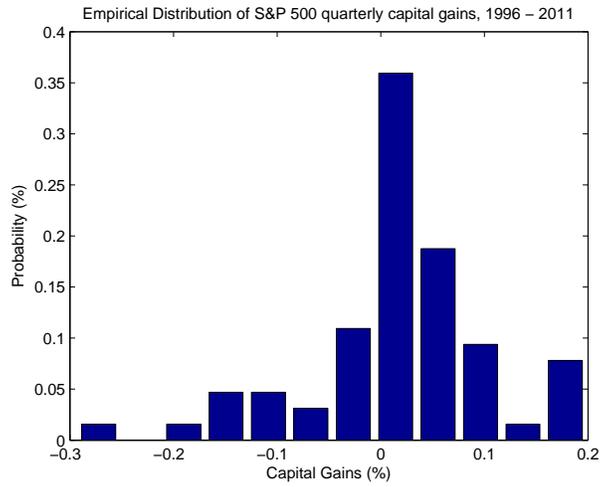
Implications for physical probabilities and risk adjustments in the economic model using consumption growth states. Top panel shows unconditional probability of bad states recovered in the model and the value of the stochastic discount factor from good to bad state relative to staying in a good state as a function of the preference parameter θ . Bottom panel shows the implied average market return and the Kullback-Leibler divergence between the implied and calibrated physical probabilities as a function of the preference parameter θ . The dashed line represents the value given by θ that minimizes the Kullback-Leibler (KL) divergence. The output is based on the economic model.

Figure 2: Probabilities and Risk Adjustments: Economic Model, Return States



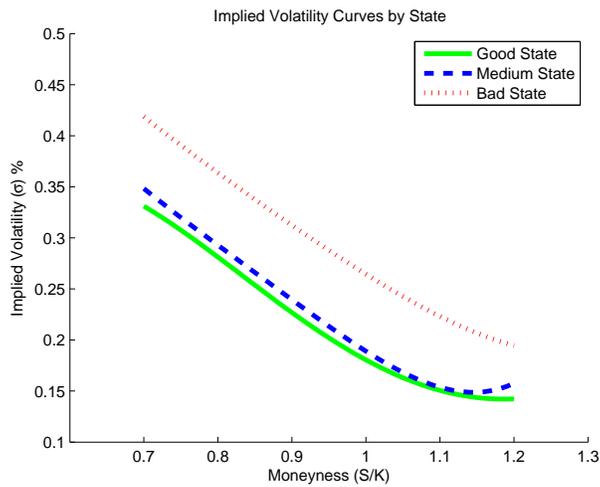
Implications for physical probabilities and risk adjustments in the economic model using return states. Top panel shows unconditional probability of bad states recovered in the model and the value of the stochastic discount factor from good to bad state relative to staying in a good state as a function of the preference parameter θ . Bottom panel shows the implied average market return and the Kullback-Leibler divergence between the implied and calibrated physical probabilities as a function of the preference parameter θ . The dashed line represents the value given by θ that minimizes the Kullback-Leibler (KL) divergence. The output is based on the economic model.

Figure 3: Empirical Distribution of Market Capital Gains



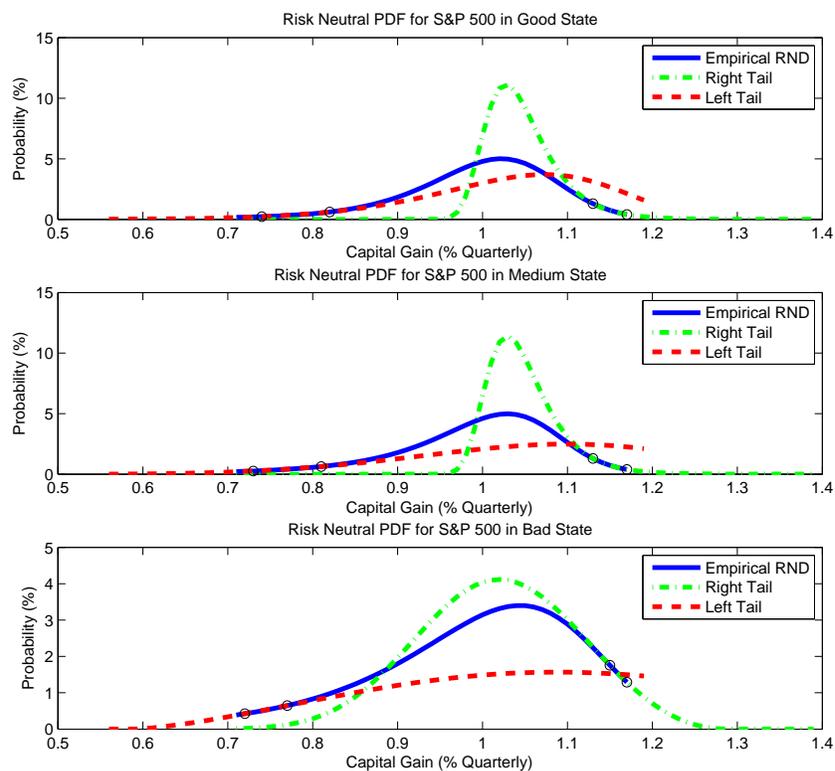
Empirical distribution of log capital gains on S&P 500. Quarterly observations from 1996 to 2011.

Figure 4: Implied Volatility Curves in Economic States



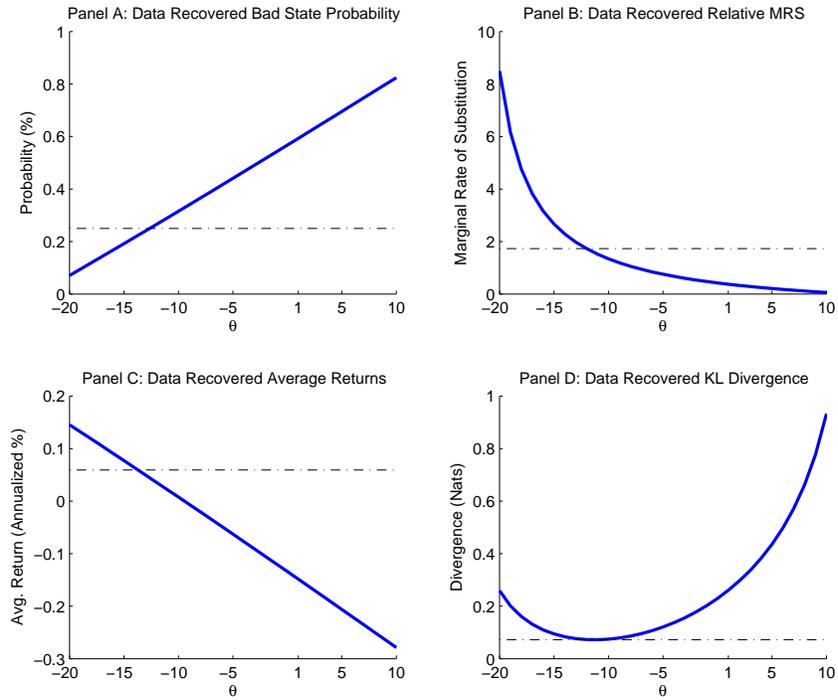
Implied volatility curves for a range of moneyness (spot/strike) in each aggregate economic state. Quarterly observations from 1996 to 2011.

Figure 5: Recovered Risk-Neutral Densities in Economic States



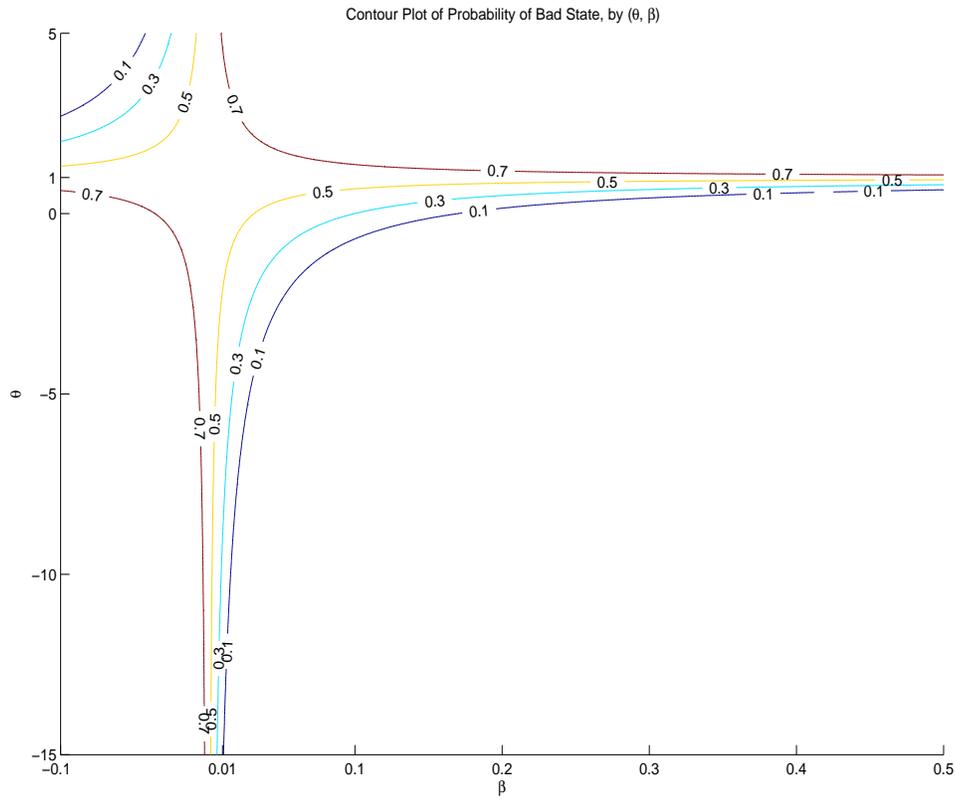
Empirical risk-neutral densities for the market capital gains in good, medium and bad economic states. The blue solid line represents the portion constructed from option data alone; red dashed and green dashed-dotted lines represent left and right tails, respectively, constructed using the GEV approximation to the underlying data density. Quarterly observations from 1996 to 2011.

Figure 6: Probabilities and Risk Adjustments: Data



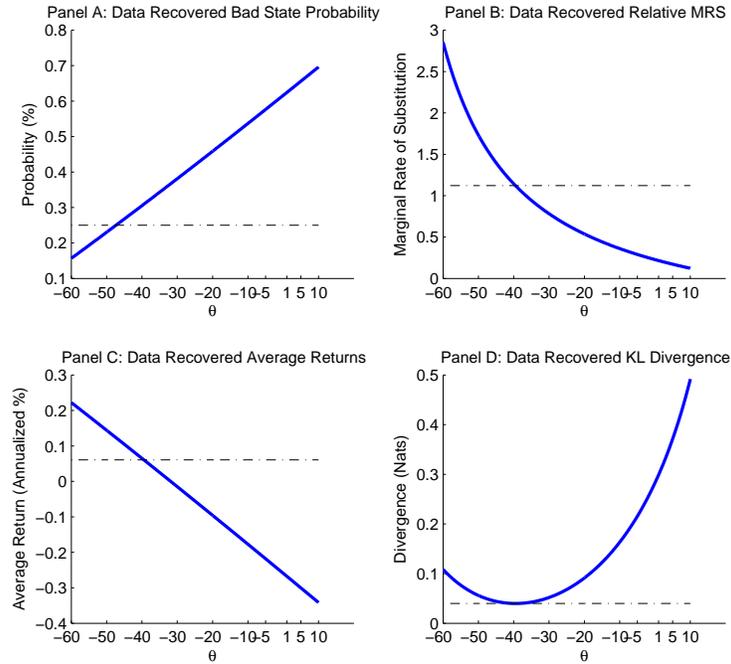
Implications for physical probabilities and risk adjustments in the data. Top panel shows unconditional probability of bad states recovered in the model and the value of the stochastic discount factor from good to bad state relative to staying in a good state as a function of the preference parameter θ . Bottom panel shows the implied average market return and the Kullback-Leibler divergence between the implied and calibrated physical probabilities as a function of the preference parameter θ . The dashed line represents the value given by θ that minimizes the Kullback-Leibler (KL) divergence. The output is based on quarterly observations from 1996 to 2011.

Figure 7: Implied Bad State Probability



Contour plot of recovered bad state probability in the data, for different combinations of preference parameters θ (y-axis) and wealth-to-consumption scale factor β (x-axis). Quarterly observations from 1996 to 2011.

Figure 8: Implications for Probabilities and Risk Adjustments: Monthly Data



Top panel shows unconditional probability of bad states recovered in the model and the value of the stochastic discount factor from good to bad state relative to staying in a good state as a function of the preference parameter θ . Bottom panel shows the implied average market return and the Kullback-Leibler divergence between the implied and calibrated physical probabilities as a function of the preference parameter θ . The dashed line represents the value given by θ that minimizes the Kullback-Leibler (KL) divergence. The output is based on monthly horizons at primary cycle expirations (March - June - September - December) from 1996 to 2011.